Hypersphere space-time model

(draft)

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Abstract

The origin of the three spatial dimensions as well as that of time is deduced from fundamental principles (symmetry, homogeneity). The structure resulting from this construction looks like an hypersphere of which each energy particle constitutes a dimension. This model shall be linked to the existing theories that are in adequation with the experience.

Generation of space

Supposing that nothing (*symmetry*) generates something (*energy*), an energy quantum (a) and its opposite ($\overline{a}$) can be formalized like this :

$a + \overline{a} = 0$ (*symmetry*), $a \times \overline{a} = 1$ (*energy*) $\rightarrow$ $a = i$ and $\overline{a} = -i$ where $i^2 = -1$.

The quantum (a) is a complex number ($a = a_1+ia_2 \in \mathbb{C}$, a_1, $a_2 \in \mathbb{R}$, $i^2 = -1$) so it behaves like a wave, more precisely like the π/2 phase of a virtual (potential) standing wave. It's the same for the opposite ($\overline{a}$).

The quantum (a) and its opposite ($\overline{a}$) form a pair of complex numbers ($a,\overline{a}$), commonly referred as a spinor or as a quaternion.

A quaternion (q) is defined by

$q = a + \overline{a}j = a_1 + ia_2 + \overline{a}_1 j + i\overline{a}_2 j = a_1 + ia_2 + j\overline{a}_1 + k\overline{a}_2$ where $j^2 = -1$, $ij = k = -ji$,

or in a more general notation

$q = s + ix + jy + kz$ where $s, x, y, z \in \mathbb{R}$, $i^2=j^2=k^2=ijk=-1$,

or in another way

$q = s + \hat{\imath}v$ where $\hat{\imath}^2 = -1$,

s is the scalar part and $v = (x,y,z)$ the 3 dimensional vector part.

The result of the product of quaternions (q) and (q') is also a quaternion

$qq' = (s+\hat{\imath}v)(s'+\hat{\imath}v') = (ss' - v.v') + \hat{\imath}(sv' + s'v + v\times v')$

where the scalar part is $(ss' – v.v')$ and the vectorial part is $(sv' + s'v + v\times v')$, $v.v' = xx'+yy'+zz'$ is the scalar product of vectors and $v\times v' = (yz'-y'z, zx'-z'x, xy'-x'y)$ the vectorial product of vectors. The product of quaternions is not commutative because the vectorial product is anti-commutative ($v\times v' = -v'\times v$).

Here are some definitions in relation to quaternions :

conjugate ($\overline{q}$) of the quaternion (q)
$\overline{q} = s - \hat{\imath}v = s - ix - jy - kz$

bra-ket product <|>
$<q|q'> = q\overline{q'}$
$= (s+\hat{\imath}v)(s'-\hat{\imath}v') = ss'+v.v'+\hat{\imath}(-sv'+s'v-v\times v') = (ss'+v.v')-\hat{\imath}(-s'v+sv'-v'\times v)$
$= \overline{<q'|q>}$

quadratic form over (q), also called the norm
$||q||^2 = <q|q> = q\overline{q} = ss + v.v + \hat{\imath}(sv - sv - v\times v) = s^2 + ||v||^2 = s^2 + x^2 + y^2 + z^2 \in \mathbb{R}$

not commutative division by (q), only if (q) ≠ 0
$1/q = \overline{q}/||q||$

orthogonality between (q) and (q')

$$||q+q'||^2 = ||q||^2+||q'||^2+\langle q|q'\rangle+\langle q'|q\rangle = ||q||^2+||q'||^2 \quad \rightarrow \quad \langle q|q'\rangle+\langle q'|q\rangle = 0$$

Orthogonality is a relation between two elements (q) and (q'), implying

$$\langle q|q'\rangle + \langle q'|q\rangle = 0$$
$$\rightarrow \quad ss' + v.v' + \hat{\imath}(s'v - sv' - v\times v') + s's + v'.v + \hat{\imath}(sv' - s'v - v'\times v) = 0$$
$$\rightarrow \quad ss' + v.v' + s's + v'.v = 2(ss' + v.v') = 0$$
$$\rightarrow \quad ss' + v.v' = 0$$

because v.v' = v'.v and v×v' = -(v'×v).

If (v) and (v') are orthogonal (v.v' = 0), (s) or (s') shall be null to keep orthogonality of quaternions.

According to the *homogeneity* principle, all quaternions issued by the (a,$\bar{a}$) quanta are identicals and are supposed to be unitary (||q|| = 1). Such a quaternion is here called a **quantion**. A quantion has a three dimensional (3D) vectorial part only which is a velocity vector. The quantions are the 3D space but each quantions has its own 3D space referential. That's the generation of 3D space.

By the far from evidence hypothesis that energy is constant, so finite, there is a constant number N of quantions in the universe. If quantions are indistinguishable, they are independant to each others, so orthogonal. Then they form the surface of an hypersphere in a vectorial space of size N, corresponding to the number of quantions.

$$N = \sum ||q||^2 = \sum (s^2 + x^2 + y^2 + z^2)$$

Each energy quantum wave (a) has the same amplitude and same frequency and cover the whole universe on its dimension, so half of the wavelength of (a) can be considered as the size of the universe.

The total energy of the universe is the bra-ket multiplication (volume) of all quantions.

Position in space

A set of quantions can be projected on a direction to form a particle. The multiplication of (n) quantions forms a frequency wave of (n).

$$q^n \rightarrow \text{wave of frequency (n)}$$

But there is others solutions if (n) has a divider (m)

$$mq^{n/m} \rightarrow \text{sum of m waves of frequency (n/m)}$$

So the sum of possibilities is

$$\Sigma\, n_i q^{n_i}$$

It's a kind of Fourier serie where the sum of waves generates a more accurate position in the space.

Generation of time

Quantions are expressed as virtual standing waves with π/2 phase. Even if a wave has a frequency that implies time, everything is static, without change, because of the standing state. Time is generated from change and from causality, that orders the change.

The only thing that could change is the perspective of the observation, the projection on the observer's perspective. A projection is a removal of one or more dimensions.The change of perspective changes the direction of particles or creates particles or destroys particles in the observer's reference. A projection is an interaction.

Causality

The bra-ket product is usually not associative

$$<<q|q'>|q''> = <q\overline{q}'|q''> = q\overline{q}'\overline{q}''$$
$$<q|<q'|q''>> = <q|q'\overline{q}''> = qq''\overline{q}'$$

The bra-ket product introduces the causality by ordering operations, only if there is a scalar part in orthogonal quantions (q = s + îv, s ≠ 0) because if q = îv, q' = îv', q'' = îv''

$$<<\hat{\imath}v|\hat{\imath}v'>|\hat{\imath}v''> = (\hat{\imath}v)(-\hat{\imath}v')(-\hat{\imath}v'') = -\hat{\imath}(v\times v'\times v'')$$
$$<\hat{\imath}v|<\hat{\imath}v'|\hat{\imath}v''>> = (\hat{\imath}v)\overline{(\hat{\imath}v')(-\hat{\imath}v'')} = (\hat{\imath}v)(\hat{\imath}v'')(-\hat{\imath}v') = \hat{\imath}(v\times v''\times v') = -\hat{\imath}(v\times v'\times v'')$$

Time

The quantions as unitary quaternions are isomorph to SU(2) Lie group.

1 replacing the Gell-Mann matrix diag(1,1,-2) by the two matrices diag(1,0,-1) and diag(0,1,-1), forming generators of U(3) where each generator is one generator of iSU(2), then U(3) = iSU(2)×iSU(2)×iSU(2)

$$\begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix}\begin{pmatrix}0&-i&0\\i&0&0\\0&0&0\end{pmatrix}\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix}\rightarrow i\,\mathrm{SU}(2)$$

$$\begin{pmatrix}0&0&1\\0&0&0\\1&0&0\end{pmatrix}\begin{pmatrix}0&0&-i\\0&0&0\\i&0&0\end{pmatrix}\begin{pmatrix}1&0&0\\0&0&0\\0&0&-1\end{pmatrix}\rightarrow i\,\mathrm{SU}(2)$$

$$\begin{pmatrix}0&0&0\\0&0&1\\0&1&0\end{pmatrix}\begin{pmatrix}0&0&0\\0&0&-i\\0&i&0\end{pmatrix}\begin{pmatrix}0&0&0\\0&1&0\\0&0&-1\end{pmatrix}\rightarrow i\,\mathrm{SU}(2)$$

2 U(3) = SU(3)×U(1)
3 from (1) and (2), SU(3)×SU(2)×U(1) = -i (SU(2)×SU(2)×SU(2)×SU(2))
4 SU(3)×SU(2)×U(1)×iSU(2) = SU(2)×SU(2)×SU(2)×SU(2)×SU(2)

The standard model SU(3)×SU(2)×U(1) is based on a broken composition of SU(2). The additionnal iSU(2) in (4) can be the 3 higgs bosons for the W and Z bosons, as well as the 3 higgs bosons for the 3 families of mass of fermions. The higgs bosons introduce a scalar part in the quantions.

The scalar part is a complex number.

Consequences

Field

The concept of field is immediate. It's the quantion’s wave envolving the whole universe.

Entaglement

Entaglement is a particle that is the addition of two other ones. Interaction is applyed simultanously on the two particles.

$$q = q_1 + q_2 \quad \rightarrow \quad <q'|q> = <q'|q_1> + <q'|q_2>$$

Antimatter

Matter and antimatter annihilation generates energy. Bra-ket multiplication of matter particle (q) with antimatter particle (q^*) has to generate vectorial particle (îv’) in a collision ($v = v^*$)

$$<q|q^*> = ss^* + v.v^* + î(s^*v - sv^* - v \times v^*) = îv' \neq 0$$

$\rightarrow \quad ss^* = v.v^*$

$\rightarrow$

Shape of the universe

Quantions are orthogonals in their interaction, with an euclidian norm. Then the corresponding space-time structure is euclidian, so fundamentally flat.

Relativity

Theory agrees with the special relativity (Lorentz scalar) because of its fit with the Minkowski formula : $x^2+y^2+z^2+s^2 = c^2t^2$. Each elementary element, the quantion, has its own 3D reference space as required by the special relativity.

According to the above special relativity, gravity is generated by the 'mass' dimension (s) that decreases space (x,y,z) for same time (t), which agrees with the slowing down because of the mass in the general relativity. The general relativity is statistically generated from quantions and is not a fundamental structure of space-time. The number of quantions influences the behaviour of gravitation.

Standard Model

The standard model SU(3)×SU(2)×U(1) is based on a composition of SU(2).

There is no SU(n>2) group from composition of SU(2) groups because :

1 $\dim(SU(n)) = n^2 - 1 \quad \rightarrow \quad \dim(SU(2)) = 3$
2 if $SU(x) = SU(m) \times SU(n) \quad \rightarrow \quad \dim(SU(x)) = \dim(SU(m)) + \dim(SU(m))$
3 if $SU(n) = SU(2) \times \ldots \times SU(2)$
$\rightarrow \dim(SU(n)) = \dim(SU(2)) + \ldots + \dim(SU(2)) = 3 + \ldots + 3$
$\rightarrow$ dim(SU(n)) = number of SU(2) generator matrices for covering SU(n) generators matrices
$\rightarrow \dim(SU(n)) \geq n \times (n-1) / 2$
4 from (1) and (3)
$\rightarrow (n^2 - 1) / 3 \geq n \times (n-1) / 2$
$\rightarrow \quad n \leq 2$

5 if $n > 2$, then $SU(n) \neq SU(2) \times \ldots \times SU(2)$

The same way can be used to demonstrate that :

- $U(N) = SU(N)\times U(1) \neq SU(2)\times\ldots\times SU(2)$ if $N > 3$
- $SU(N_1)\times SU(N_2)\times\ldots \neq SU(2)\times\ldots\times SU(2)$ if one $N_i > 3$

So, there is no other interaction than the ones of the standard model.

Unitary electric charge

If everything looks like a rotation of π/2 phase, the negative electric charge can be defined by a phase shift of -π/2 (charge -1). **Unit electric charges** could be used for up quark with shift of 2π/2 (charge +2) and down quark with shift of π/2 (charge +1) because $4\pi/2 \equiv 0$.

- up+up+down = (2+2+1) π/2 ≡ 1 π/2 (proton)
- up+down+down = (2+1+1) π/2 ≡ 0 π/2 (neutron)

By this way, a symmetry could be established between the charge of leptons (electron -1, neutrino 0) and the one of quarks (down +1, up +2).

Conclusion

Based on the hope that Nature is simple, this article introduces a new representation of space-time structure of the universe : an hypersphere structure on a multi-dimensional space, each dimension is an energy quantum with its opposite forming a quaternion covering the whole universe.

There is still a long way to envolve the whole physic in one theory but this bottom-up approach, from simple principles to more complex structures, in adequation with the observed reality, is probably a good way to elaborate a simple and comprehensive theory. This intuitive approach tries to answer to a fundamental question : why has the universe an apparent 3 dimensional structure in addition of time, which is far from an evidence ?

To explain the universe, the ether is not necessary, perhaps neither is space-time.

Puzzle :
Position = temps. La position nait d'une addition de fréquences potentielles (multiple de nombres premiers).
Projection = rotation
Conservation de la quantité de mouvement à analyser
pourquoi addition pour intrication ?
Quand (temps?) et pourquoi brisure. Quand position et interaction. Fréquence de changement pour masse.
reverse (q̃) or mirror image of the quaternion (q) : q̃ = a + jā = s + ix + jy - kz
L'intrication est l'addition de deux particules
L'antimatière est le négatif d'une particule (valable pour is).
L'intrication est liée à la chiralité (sens) qui existe ou non entre particules. L'interaction casse-t-elle l'intrication ?
L'univers serait né en chiralité gauche uniquement (formule quaternion).
La masse permet d'inverser la chiralité sans passer à l'anti-particule (inversion du s?).
Les fermions et leur masse sont liés au bi-spineur de Dirac. Comment (pourquoi) un bi-spineur existe-t-il ? Ils existent car un paramètre est ajouté, la masse. Le bi-spineur est indispensable à l'interaction.
L'intensité de la masse dépend de la fréquence du changement de chiralité, causé par le boson de higgs ou les bosons de higgs car trois masses sont possibles. Pourquoi la masse interagit-elle surtout avec le boson de higgs le moins intrusif ? Est-ce l'énergie – cinétique – de la particule qui joue ?
L'important est de déterminer un temps entre les interactions. Les interactions marquent l'écoulement du temps (variable selon la « distance ») car le système potentiel (ondes) est stationnaire. La position peut être déterminée par la formule de Fourier. La multiplication braket n'est pas associative et garantit la causalité si s ≠ 0 (masse ≠ 0).
Il n'y a pas de perte d'information, la multiplication braket la préserve, pas l'addition ou multiplication classique. Le principe d'unitarité implique la conservation de l'information. L'interaction crée une information réelle à partir d'une information virtuelle (probabilité). L'interaction est lié à un opérateur hermitien.
La question principale est pourquoi on arrive à une interaction. L'interaction ponctuelle n'est pas possible dans un espace 3D (improbable) mais si l'espace n'existe pas ou s'il s'agit de champs, c'est bon. La masse semble indispensable à l'existence d'un interaction. Les particules intermédiaires permettent l'interaction malgré le principe d'exclusion de Pauli. L'interaction est une projection, donc une perte de dimension(s). Peut-on considérer que la probabilité est le résultat de l'interaction et non l'inverse ?

- *Indices :*
  - *interaction*
    - *position <> fréquence (Fourier)*
    - *localité = Lagrangien (scalaire) qui dépend de la position et dérivée*
    - *vitesse (constante structure fine), probabilité, intrication, particule, effondrement*
    - *section efficace = probabilité d'interaction*
    - *résonance : apport exact d'énergie pour changer l'état*
    - *diminution probabilité avec augmentation espace, impossible si ponctuel*
    - *pas pour intervalle genre espace (commutativité possible)*
    - *pas possible ponctuellement car rencontre de points improbable → champs*
    - *information conservée (Liouville) → multiplication (premiers), non associativité (causalité)*

- *propagation :*
  - *norme,*
  - *2D*
  - *impulsion = mv = énergie*
- *intrication*
  - *= concept de particule*
  - *= gravitation ?*
- *entropie :*
  - *= # modifications micros sans changement d'état macro*
  - *→ lié au nombre de dimensions ?*
  - *minimale à l'origine (orthogonalité ?)*
  - *radioactivité*
  - *chiralité gauche ?*
  - *minimale si système statique (contradiction ?)*
  - *peut dépendre de l'observateur ?*
- *bi-spineur de Dirac (ℂ×ℍ),*
  - *comment-pourquoi ?*
  - *ce qui devrait être lié au temps, à la masse, à la particule → pas d'inverse pour fermion*
  - *→ antimatière*
- *i SU2 x i SU2 x i SU2 = U3*
  - *-i introduit une rotation dans la masse (linéairement indépendant)*
  - *apport avec produit de dirac, le boson de Higgs imaginaire (higgs aussi iSU2 pour 3 familles) ?*
  - *SU3 x SU1 chiralité droite aussi, lié à charge électrique ?*
- *s :*
  - *s^2 ~ m possible,*
  - *inversion chiralité q avec s,*
  - *constante (prq),*
  - *alternance chiralité = interaction higgs = multiplication par -1*
  - *produit braket non associatif si s ≠ 0*
- *introduction de c, comment pourquoi*
- *étudier collision dans chaque point de vue (q,q') → $q'q\overline{q}'$ et $qq'\overline{q}$*
- *<|> pas associatif (temps) si le résultat est un complexe (boson higgs)*
- *lien entre x (s^2-v^2+...) et <|> (s^2+v^2) → Lagrangien et énergie ?*
- *temps t augmente (espace augmente) si v (vitesse) ou s (gravité) augmente*
- *toujours 3D ou une dimension peut disparaître ?*
- *tenir compte de l'impulsion*
- *distance = probabilité de rencontrer une particule virtuelle ?*
- *Brisure C aussi pour quarks ?*
- *Axe réel = énergie cinétique, axe imaginaire = énergie potentielle*
- *addition : perte d'information par addition, probabilité (tout ce qui est possible), dérivée de la multiplication, position si addition d'ondes d'énergie (Fourier), change module, évolution unitaire*
- *multiplication : interaction, garde information grâce aux nombres premiers mais la non commutativité perd l'information (non!), change phase, mesure*
- *C = P, presque car T joue un rôle (symétrie CPT) ?*
- *angle Weigner définit masse bosons W et Z : Mw = cos Weigner x Mz, définit aussi complage avec Z (voir livre LHC p55)*
- *Particule droite/gauche selon isospin nul ou non*

- *théorie renormalisable : aucune constante de couplage de dimension puissance négative de la masse*
- *neutrino : pseudovecteur (produit vectoriel), pourquoi pas de charge, =2 ?*
- *quaternion as exponential (exp(ivt) où v vecteur) -> exp(exp(ix)) ? -> exp(-x^2) ?*
- *reflet miroir = rotation dans dimension supplémentaire*
- *analyser changement de perspective*
- *$\overline{q}$= -1/2 (q+iqi+jqj+kqk) → utile ? bof*