Hypersphere space-time model

(draft)

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Abstract

The origin of the three spatial dimensions as well as that of time is deduced from fundamental principles (symmetry, homogeneity). The structure resulting from this construction looks like an hypersphere of which each energy particle constitutes a dimension. This model shall be linked to the existing theories that are in adequation with the experience.

Generation of space

Supposing that nothing (*symmetry*) generates something (*energy*), an energy quantum (a) and its opposite ($\overline{a}$) can be formalized like this :

$a + \overline{a} = 0$ (*symmetry*), $a \times \overline{a} = 1$ (*energy*) → $a = i$ and $\overline{a} = -i$ where $i^2 = -1$.

The quantum (a) is a complex number ($a \in \mathbb{C}$) so it behaves like a wave, more precisely like the π/2 phase of a virtual (potential) standing wave. The wavelength is the whole universe. It's the same for the opposite ($\overline{a}$).

The quantum (a) and its opposite ($\overline{a}$) form a pair of complex numbers (i,-i), commonly referred as a spinor or as a quaternion.

A quaternion (q) is defined by

$q = s + i'x + j'y + k'z = s + iv$ where $s, x, y, z \in \mathbb{R}$, $i^2=i'^2=j'^2=k'^2=i'j'k'=-1$,

(v) = (x,y,z) is called the 3 dimensional vector part and (s) is the scalar part or the quaternion.

The result of the product of quaternions (q) and (q') is also a quaternion

$qq' = (s+iv)(s'+iv') = (ss' - v.v') + i(sv' + s'v + v\times v')$

where the scalar part is $(ss' – v.v')$ and the vectorial part is $(sv' + s'v + v\times v')$, $v.v' = xx'+yy'+zz'$ is the scalar product of vectors and $v\times v' = (yz'-y'z, zx'-z'x, xy'-x'y)$ the vectorial product of vectors. The product of quaternions is not commutative because the vectorial product is anti-commutative ($v\times v' = -v'\times v$).

Here are some definitions in relation to quaternions :

conjugate ($\overline{q}$) of the quaternion (q)
$\overline{q} = s – iv = s - i'x - j'y - k'z$

bra-ket product <|>
$\langle q|q'\rangle = q\overline{q}'$

quadratic form over (q), also called the norm
$||q||^2 = \langle q|q\rangle = q\overline{q} = ss + v.v + i(sv – sv – v\times v) = s^2 + ||v||^2 = s^2 + x^2 + y^2 + z^2 \in \mathbb{R}$

orthogonality between (q) and (q')
$||q+q'||^2 = ||q||^2+||q'||^2+\langle q|q'\rangle+\langle q'|q\rangle = ||q||^2+||q'||^2$ → $\langle q|q'\rangle+\langle q'|q\rangle = 0$

Note that $q^2 = qq = (s^2 - ||v||^2) + 2i(v)$ is not a quadratic form over a quaternion because of the resulting vectorial part (v).

Orthogonality is a relation between two elements (q) and (q'), implying

$$\langle q|q'\rangle+\langle q'|q\rangle = ss' + v.v' + i(s'v - sv' - v\times v') + s's + v'.v + i(sv' - s'v - v'\times v) = 0$$
$$\rightarrow \; ss' + v.v' = 0$$

because $v.v' = v'.v$ and $v\times v' = -(v'\times v)$.

According to the *homogeneity* principle, all quaternions issued by the $(a,\bar{a})$ quanta are identicals and are supposed to be unitary ($||q|| = 1$). Such a quaternion is here called a **quantion**. A quantion has a three dimensional vectorial part only which is a velocity vector. That's the generation of space.

By the far from evidence hypothesis that energy is constant, so finite, there is a constant number N of quantions in the universe. If quantions are independant and indistinguishable, they are orthogonal and they form the surface of an hypersphere in a vectorial space of size N, corresponding to the number of quantions. This is the supposed structure of the universe.

$$N = \sum ||q||^2 = \sum (s^2 + x^2 + y^2 + z^2)$$

All the energy (E) of the universe is relative to the multiplication of all quantions.

$$E = \underbrace{q \times \ldots \times q}_{N}$$

Energy can be devided into sub parts according to prime numbers of N. Each prime number can be seen as a possible set of quantions equivalent to a particle type.

Generation of time

Quantions are expressed as virtual standing waves with π/2 phase. Even if a wave has a frequency that implies time, everything is static, without change, because of the standing state.

The only thing that could change is the perspective of the observation, the projection on the observer's perspective. The change of perspective changes the direction of particles or creates particles or destroys particles.

The quantions as unitary quaternions are isomorph to SU(2) Lie group. The standard model SU(3) x SU(2) x U(1) can be based on a composition of SU(2) :

1. replacing the Gell-Mann matrix diag(1,1,-2) by the two matrices diag(1,0,-1) and diag(0,1,-1), forming generators of U(3) where each generator is one generator of iSU(2), then U(3) = iSU(2) x iSU(2) x iSU(2)

$$\begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix}\begin{pmatrix}0&-i&0\\i&0&0\\0&0&0\end{pmatrix}\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix}\rightarrow i\,\mathrm{SU(2)}$$

$$\begin{pmatrix}0&0&1\\0&0&0\\1&0&0\end{pmatrix}\begin{pmatrix}0&0&-i\\0&0&0\\i&0&0\end{pmatrix}\begin{pmatrix}1&0&0\\0&0&0\\0&0&-1\end{pmatrix}\rightarrow i\,\mathrm{SU(2)}$$

$$\begin{pmatrix}0&0&0\\0&0&1\\0&1&0\end{pmatrix}\begin{pmatrix}0&0&0\\0&0&-i\\0&i&0\end{pmatrix}\begin{pmatrix}0&0&0\\0&1&0\\0&0&-1\end{pmatrix}\rightarrow i\,\mathrm{SU(2)}$$

2. U(3) = SU(3) x U(1)
3. from (1) and (3), SU(3) x SU(2) x U(1) = -i (SU(2) x SU(2) x SU(2) x SU(2))

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Consequences

Field

The concept of field is immediate. It's the quantion's waves envolving the whole universe.

Antimatter

Matter and antimatter annihilation generates energy. Multiplication of matter particle with antimatter particle has to generate photon.
TO BE EXPLORED
principe d'exclusion non valable, quelle variable est différente, s ?

Shape of the universe

Quantions are orthogonals in their interaction, with an euclidian norm. Then the corresponding space-time structure is euclidian, so fundamentally flat.

Relativity

Theory agrees with the special relativity (Lorentz scalar) because of its fit with the Minkowski formula : $x^2+y^2+z^2+s^2=c^2t^2$. Each elementary particle has its own 3D reference space as required by the special relativity.

According to the above special relativity, gravity is generated by the 'mass' dimension (s) that decreases space (x,y,z) for same time (t), which agrees with the slowing down because of the mass in the general relativity. It's a kind of equivalence principle. Note that the general relativity is here statistically generated from a lot of quantions.

Entaglement

Entaglement of two particles exists when the two particles are not independant (orthogonal) to each other.
TO BE EXPLORED

Standard Model

The standard model SU(3) x SU(2) x U(1) is based on a composition of SU(2).

There is no SU(n>2) group from composition of SU(2) groups because :

1. $\dim(SU(n)) = n^2 - 1$
   → $\dim(SU(2)) = 3$
2. if $SU(x) = SU(m) \times SU(n)$
   → $\dim(SU(x)) = \dim(SU(m)) + \dim(SU(m))$
3. if $SU(n) = SU(2) \times \ldots \times SU(2)$
   → $\dim(SU(n)) = \dim(SU(2)) + \ldots + \dim(SU(2)) = 3 + \ldots + 3$
   → $\dim(SU(n))$ = number of SU(2) generator matrices for covering SU(n) generators matrices
   → $\dim(SU(n)) \geq n \times (n-1) / 2$

4. from (1) and (3)
   → $(n^2 - 1) / 3 \geq n \times (n-1) / 2$
   → $n \leq 2$
5. if $n > 2$, then $SU(n) \neq SU(2) \times \ldots \times SU(2)$

The same way can be used to demonstrate that :

- $U(N) = SU(N) \times U(1) \neq SU(2) \times \ldots \times SU(2)$ if $N > 3$
- $SU(N_1) \times SU(N_2) \times \ldots \neq SU(2) \times \ldots \times SU(2)$ if one $N_i > 3$

So, there is no other interaction than the ones of the standard model.

Unitary electric charge

If everything looks like a rotation of π/2 phase, the negative electric charge can be defined by a phase shift of -π/2 (charge -1). **Unit electric charges** could be used for up quark with shift of 2π/2 (charge +2) and down quark with shift of π/2 (charge +1) because 4π/2 ≡ 0.

- up+up+down = (2+2+1) π/2 ≡ 1 π/2 (proton)
- up+down+down = (2+1+1) π/2 ≡ 0 π/2 (neutron)

By this way, a symmetry could be established between the charge of leptons (electron -1, neutrino 0) and the one of quarks (down +1, up +2).

Conclusion

Based on the hope that Nature is simple, this article introduces a new representation of space-time structure of the universe : an hypersphere structure on a multi-dimensional space, each dimension is an energy quantum with its opposite.

There is still a long way to envolve the whole physic in one theory but this bottom-up approach, from simple principles to more complex structures, in adequation with the observed reality, is probably a good way to elaborate a simple and comprehensive theory. This intuitive approach tries to answer to a fundamental question : why has the universe an apparent 3 dimensional structure in addition of time, which is far from an evidence ?

To explain the universe, the ether is not necessary, perhaps neither is space-time.

Puzzle :

- *TODO :*
  - *vérifier s ~ m : tester valeur, $E^2/c^2 = p^2 + m^2c^2 \rightarrow p = mv = , mc^2=hv$*
  - *antimatière x matière (spin 1/2) = photon → $s^2 + v.v' = 0$*
  - *spin & chiralité : définition, différence avec antiparticule*
- *Indices :*
  - *interaction*
    - *position <> fréquence (Fourier)*
    - *localité = Lagrangien (scalaire) qui dépend de la position et dérivée*
    - *résonance, vitesse (constante structure fine), probabilité, intrication, particule, effondrement*
    - *diminution probabilité avec augmentation espace, impossible si ponctuel*
    - *pas pour intervalle genre espace (commutativité possible)*
  - *propagation : norme, 2D*
  - *entropie : minimale à l'origine (orthogonalité ?), radioactivité, chiralité gauche ?, minimale si système statique*
  - *bi-spineur de Dirac ($\mathbb{C}\times\mathbb{H}$), ce qui devrait être lié au temps, à la masse, à la particule → pas d'inverse pour fermion*
  - *i SU2 x i SU2 x i SU2 = U3 → -i introduit une rotation dans la masse (linéairement indépendant) → rapport avec produit de dirac, le boson de Higgs imaginaire (higgs aussi SU2 pour 3 familles) ?*
  - *s : s^2 ~ m possible, inversion chiralité q avec s, constante (prq), alternance chiralité = interaction higgs*
  - *introduction de c, comment pourquoi*
  - *étudier collision dans chaque point de vue (q,q') → $q'q\bar{q}'$ et $qq'\bar{q}$*
  - *<|> pas associatif (temps) si le résultat est un complexe (boson higgs)*
  - *lien entre x ($s^2-v^2+...$) et <|> (s^2+v^2) → Lagrangien et énergie ?*
  - *tenir compte de l'impulsion*
  - *distance = probabilité de rencontrer une particule virtuelle ?*
  - *Brisure C aussi pour quarks ?*
  - *Axe réel = énergie cinétique, axe imaginaire = énergie potentielle*
  - *addition : perte d'information par addition, probabilité (tout ce qui est possible), dérivée de la multiplication, position si addition d'ondes d'énergie (Fourier), change module, évolution unitaire*
  - *multiplication : interaction, garde information grâce aux nombres premiers mais la non commutativité perd l'information, change phase, mesure*
  - *C = P, presque car T joue un rôle (symétrie CPT) ?*
  - *angle Weigner définit masse bosons W et Z : Mw = cos Weigner x Mz, définit aussi complage avec Z (voir livre LHC p55)*
  - *Particule droite/gauche selon isospin nul ou non*
  - *théorie renormalisable : aucune constante de couplage de dimension puissance négative de la masse*
  - *neutrino : pseudovecteur (produit vectoriel), pourquoi pas de charge, =2 ?*
  - *quaternion as exponential (exp(ivt) où v vecteur) -> exp(exp(ix)) ? -> $exp(-x^2)$?*
  - *reflet miroir = rotation dans dimension supplémentaire*
  - *analyser changement de perspective*
  - *$\bar{q}$= -1/2 (q+iqi+jqj+kqk) → utile ? bof*