Hypersphere space-time model

(draft)

Abstract

Following some fundamental principles of physic (symmetry, homogeneity, ...), the article proposes a simple model for the structure of the universe. Time is no more a fundamental dimension, it's the result of the 3 spatial dimensions, in addition to one dimension for mass. Furthermore, there is no 4 fundamental dimensions but as much dimensions as energy particles or energy quanta, grouped into the 4 above dimensions. The universe is like an hypersphere of quanta. Each quantum behaves like a wave. Our 3 dimensional space appearance is the result of interaction between quanta. The article tries to link this model to the existing theories that are in adequation with the experience.

Generation of 3D space

Quaternion

Supposing that nothing (*symmetry*) generates something (*energy*), we can formelize an energy quantum (a) and its opposite ($\overline{a}$) like this :

$a + \overline{a} = 0$ (*symmetry*), $a \times \overline{a} = 1$ (*energy*), so $a = i$ *and* $\overline{a} = -i$ where $i^2 = -1$.

So, there are only two operations : addition (+) and multiplication (× or no sign for adequation with the common notation).

The quantum (a) is a complex number so it behaves like a wave, more precisely like the π/2 phase of a virtual (potential) standing wave. It's the same for the opposite ($\overline{a}$), so the quantum (a) and its opposite form a pair of complex numbers (a,$\overline{a}$), commonly referred as a spinor or as a quaternion.

A quaternion (q) is a vectorial space of four dimensions : one *scalar* dimension (s) and three *vectorial* dimensions (x,y,z).

$q = s + v$ where $v = (x,y,z) = ix + jy + kz$ and $s, x, y, z \in \mathbb{R}$
                  and $ij = -ji = k$, $i^2 = j^2 = k^2 = -1$

The conjugate ($\overline{q}$) of a quaternion (q) is the same quaternion with the opposite vector

$\overline{q} = s - v$

Each (a,$\overline{a}$) quaternion can be defined by one of the following formula

$a + j\,\overline{a}$ or $a + \overline{a}\,j$ where

Following the above formula, there is two kinds of quaternion : left or right. So we have the two following quaternions

$i + j(-i) = i + k$ or $i + (-i)j = i + k$

and multiplying two or more quaternions gives

$(i-k)(i+k) = 2j = -(i+k)(i-k)$
$(i-k)(i-k) = -2 = (i+k)(i+k)$
$(i-k)j = k+i = -j(i-k)$
$(i+k)j = k-i = -j(i+k)$

For convenience, we can right multiply the two quaternions by $(1+j)/2$ to get

$(i-k)(1+j)/2 = (i-k+k+i)/2 = i$ or $(i+k)(1+j)/2 = (i+k+k-i)/2 = k$

which keeps the same relations of multiplication.

Introducing the Dirac notation (<|>) which is defined by the standard multiplication with the conjugate

$$\langle a|b \rangle = a\bar{b}$$

the scalar norm ($\|q\|$) of a quaternion is a real number defined by

$$\|q\|^2 = \langle q|q \rangle = q\bar{q} = s^2 + \|v\|^2 = s^2 + x^2 + y^2 + z^2$$

Each quaternion has its own 3 dimensional reference system. According to *homogeneity* principle, there is no difference between each quaternion, so each one is supposed to be an unitary quaternion (unitary norm : $\|q\|^2 = 1$). This specific quaternion is called a *quanton* to avoid confusion with a generic quaternion.

Quantons are expressed as standing waves, so everything is static, without change nor time. The only thing that could change is the perspective of the observation, the projection on the observer's perspective.

By the far from evidence hypothesis that energy is constant, so finite, there is a constant number N of quantons.

If quantons are independant (orthogonal) to each others, they form the surface of an hypersphere in a vectorial space of size N, corresponding to the number of quantons. This is the supposed structure of the universe.

$$N = \sum \|Q\|^2 = \sum (s^2 + x^2 + y^2 + z^2)$$

so the multiplication is defined by the classical multiplication by the conjugate, so the Dirac notation defines de multiplication

$$ab \equiv \langle a|b \rangle$$

By principle, energy never disappears, so there is only multiplication and no addition in the real world.

The rearranged Minkowski formula ($s^2+x^2+y^2+z^2=c^2t^2$) looks like the surface of an hypersphere too in an euclidian space of 4 dimensions. It also looks like the norm (or the sum of norms) of quaternion. The (s) parameter is relative to mass in the Minkowski formula, so the 4th dimension is not time (t) but mass and it is relative to the scalar dimension (s) of quaternion.

An interaction is a perspective change in the 3D relations between quantons. That's why it looks like a rotation or direction change.

BUT how
- dynamism (change)
- interaction
- fermion
- Higgs mechanism

Consequences

Field

The concept of field is immediate. It's the quanton's waves envolving the whole universe.

Antimater

Mater and antimater distinction is made with left and right quaternions.

Shape of the universe

Quantons are orthogonals in their interaction, with an euclidian norm. Then the corresponding space-time structure is euclidian, so fundamentally flat.

Relativity

Theory agrees with the special relativity (Lorentz scalar) because of its fit with the Minkowski formula : $x^2+y^2+z^2+s^2=c^2t^2$.

According to the above special relativity, gravity is generated by the 'mass' dimension (s) that decreases space (x,y,z) for same time (t), which agrees with the slowing down because of the mass in the general relativity. Note that the general relativity is here statistically generated from a lot of quantons.

Entaglement

Entaglement of two particles exists when the two particles are not independant (orthogonal) to each other.

Standard Model

The quantons as unitary quaternions are isomorph to SU(2) Lie group. The standard model SU(3) x SU(2) x U(1) can be based on a composition of SU(2) :

1. replacing the Gell-Mann matrix diag(1,1,-2) by the two matrices diag(1,0,-1) and diag(0,1,-1), forming generators of U(3) where each generator is one generator of SU(2), then U(3) = SU(2) x SU(2) x SU(2)

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \to SU(2)$$

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \to SU(2) \quad \Biggr\} \quad U(3)$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \to SU(2)$$

2. U(3) = SU(3) x U(1)

3. from (2) and (3), SU(3) x SU(2) x U(1) = SU(2) x SU(2) x SU(2) x SU(2)

There is no SU(n>2) group from composition of SU(2) groups because :

1. dim(SU(n)) = n² - 1
    → dim(SU(2)) = 3
2. if SU(x) = SU(m) x SU(n)
    → dim(SU(x)) = dim(SU(m)) + dim(SU(m))
3. if SU(n) = SU(2) x … x SU(2)
    → dim(SU(n)) = dim(SU(2)) + … + dim(SU(2)) = 3 + … + 3
    → dim(SU(n)) = number of SU(2) generator matrices for covering SU(n) generators matrices
    → dim(SU(n)) ≥ n x (n-1) / 2
4. from (1) and (3)
    → (n² – 1) / 3 ≥ n x (n-1) / 2
    → n ≤ 2
5. if n > 2, then SU(n) ≠ SU(2) x … x SU(2)

The same way can be applied to prove that :

- U(N) = SU(N) x U(1) ≠ SU(2) x … x SU(2) if N > 3
- $SU(N_1)$ x $SU(N_2)$ x … ≠ SU(2) x … x SU(2) if one N_i > 3

Unitary electric charge

If everygthing looks like a rotation as π/2 phase, the negative electric charge can be defined by a phase shift of -π/2 (charge -1). **Unit electric charges** could be used for up quark with shift of 2π/2 (charge +2) and down quark with shift of π/2 (charge +1) because 4π/2 ≡ 0.

- up+up+down = 2+2+1 ≡ 1   (proton)
- up+down+down = 2+1+1 ≡ 0   (neutron)

By this way, a symmetry could be established between the charge of leptons (electron -1, neutrino -2 or 0) and the one of quarks (down +1, up +2).

Conclusion

Based on the hope that Nature is simple, this article introduces a new representation of space-time structure of the universe : an hypersphere structure on a multi-dimensional space, each dimension is an energy quantum with its opposite.

There is still a long way to envolve the whole physic in one theory but this bottom-up approach, from simple principles to more complex structures, in adequation with the observed reality, is probably a good way to elaborate a simple and comprehensive theory. This intuitive approach tries to answer to a fundamental question : why has the universe an apparent 3 dimensional structure, which is far from an evidence ?

To explain the universe, we didn't need the ether, maybe we don't need a space-time either.

Puzzle :

- *addition : perte d'information par addition, probabilité (tout ce qui est possible), dérivée de la multiplication, position si addition d'ondes d'énergie (Fourier)*
- *multiplication : garde information grâce aux nombres premiers, interaction*
- *brisure de symétrie → symétrie cachée car choix aléatoire de direction (symétrique si un grand nombre de fois)*
- *masse imaginaire boson de Higgs → chapeau mexicain*
- *fermion = bi-spineur = $\mathbb{C} \times \mathbb{H}$, → quelle modélisation ?*
- *expension de l'univers = création de fermions qui ne peuvent garder la même position*
- *Higgs = mélange (alternance) des particules gauches et droites → vitesse < c et changement de chiralité*
- *angle Weigner définit masse bosons W et Z : Mw = cos Weigner x Mz, définit aussi complage avec Z*
- *temps lié à antimatière, masse (changement hélicité), changement (de perspective), fréquence, décohérence, non-associativité, diminution de la force (électro) au carré de la distance, radioactivité*
- *radioactivité lié à entropie, effet tunnel, chiralité gauche*
- *utilisation de norme comme produit → anti-associativité, propagation (probabilité) 2D*
- *interaction si fréquences (résonance) déterminées (photon traverse ou non mur), lié à nombre premier ?*
- *théorie renormalisable : aucune constante de couplage de dimension puissance négative de la masse*
- *neutrino : pseudovecteur (produit vectoriel)*
- *quaternion as exponential (exp(ivt) où v vecteur) -> exp(exp(ix)) ? -> exp(-x²) ?*
- *modules différents avec phase sans importance équivalent à phases différentes et module = 1 ???*
- *reflet miroir = rotation dans dimension supplémentaire*
- *pseudoscalaire de Clifford = espace commun*
- *q'= -1/2 (q+iqi+jqj+kqk)*