Hypersphere space-time model

by Frédéric Paquay
2026-11-22

Abstract

The origin of the three spatial dimensions as well as the one of time can be deduced from fundamental principles (symmetry). The structure resulting from this construction looks like an hypersphere in which each energy particle constitutes a dimension, forming a loop or a string covering the whole universe. This model shall be linked to the existing theories that are consistent with the reality as measured by experiments.

Article

Space

Quantum wave

Nothing (symmetry) generating something (energy) can be expressed by the addition and the multiplication of an energy quantum (a) and its opposite ($\bar{a}$) :

$a + \bar{a} = 0$ (symmetry) $\quad a\bar{a} = 1$ (energy)

The solution is

$a = i \quad \bar{a} = -i$

where i is the common imaginary number ($i^2 = -1$). The quantum (a) is a complex number that could be considered as the $\pi/2$ phase of a virtual (potential) standing wave whose wavelength covers the whole universe. It's the same for the opposite ($\bar{a}$).

Quaternion

A quaternion ($q_x \in \mathbb{H}$) is defined by

$$q_x = x_0\, i_0 + x_1\, i_1 + x_2\, i_2 + x_3\, i_3$$

where $x_k \in \mathbb{R}$, $i_0^2 = 1$, $i_j^2 = -1$, $i_j\, i_k = -i_k\, i_j$ for j,k = 1,2,3 and $j \neq k$. The real or scalar part of the quaternion is x_0 and the vectorial or imaginary part is (x_1, x_2, x_3).

The conjugate ($\bar{q}_x$) of the quaternion (q_x) is defined by

$$\bar{q}_x = x_0\, i_0 - x_1\, i_1 - x_2\, i_2 - x_3\, i_3$$

The product of two non-zero quaternions is always non-zero. The scalar product of two quaternions (q_x) and (q_y) is defined by

$$\langle q_x, q_y \rangle = x_0\, y_0 + x_1\, y_1 + x_2\, y_2 + x_3\, y_3$$

and the norm $\|q_x\|$ as

$$\|q_x\|^2 = \langle q_x, q_x \rangle = q_x\, \bar{q}_x = x_0^2 + x_1^2 + x_2^2 + x_3^2$$

A unitary quaternion (q) is a quaternion with norm 1.

$$\|q\| = 1$$

The inverse (q^{-1}) exists for any non-zero quaternion (q), generating a division algebra

$$q^{-1} = \bar{q}/\|q\|^2$$

Field

The quantum wave (a) and its opposite can be replaced by the multiplication of two pure vectorial quaternions (q_x, $x_0 = 0$) and (q_y, $y_0 = 0$) that are orthogonal because the resulting quaternion (q_z) is pure vectorial and

$$a \equiv q_x\, q_y = q_z = -\bar{q}_z = -q_y\, q_x \equiv -\bar{a} \qquad (1)$$

$$a\, \bar{a} \equiv q_z\, \bar{q}_z = 1$$

The quaternion is a piece of momentum, a piece of energy, an energy quantum. Energy conservation implies that the result of the operation on two pairs of quanta shall not be null and can be reversed, which is in adequation with the quaternionic multiplication and division. So each fundamental element covering the whole universe can be represented by a quaternion. This simply materializes the concept of field.

Homogeneity

Considering that the universe is made of such fundamental energy elements, energy conservation implies a constant (finite) number of such elements or else an homogeneity that means that all elements are identical or even both, constant number of elements and homogeneity. Homogeneity seems to be a principle and it will be supposed.

According to the homogeneity principle, all quaternions can be considered as unitary quaternion ($\|q\| = 1$). The easiest way to explain homogeneity is to have only one element that interacts with itself in several ways but this extreme hypothesis requires further study.

Orthogonality

Each fundamental energy element has - or "is" because there is no other characteristic - its own three dimensional (3D) space, perfectly in accordance with the special relativity. Assuming that all the elements are indistinguishable implies that they are identical. Assuming that each element has its own existence implies that the elements are independant to each others and that they stand on their own dimension.

If the elements are linearly independant, there is an orthogonal representation of

them, forming an hypersphere of 3D spaces, an hypersphere of quaternions.

Particle

A particle is a subset of the hypersphere of quaternions. This subset corresponds to a volume in the structure which is the multiplication of all the elements of the subset. The volume is a quaternion.

Mass

In a first approach, mass existence could be linked to a non negative value of the scalar part ($x_0 \neq 0$) of the quaternion, the norm forming the Minkowski space $t^2 = x_0^2 + x_1^2 + x_2^2 + x_3^2$. But the quaternion is here unitary and purely vectorial ($x_0 = 0$). Furthermore, this approach doesn't answer to a lot of questions.

Mass is specific to some particles, to some set of energy, not to all kind of energy. Mass is expressed inside the Dirac equation. Mass comes from external interaction with the BEH (Higgs) boson, this interaction changes the chirality of the particle. Mass is constant at rest and is spread in 3 generations, according to some relations (e.g. CKM and PMNS matrices). Mass is annihilated by the antiparticle. All these different things shall match. The following explores some research areas but it needs to be developed deeper and more rigorously.

Special relativity

Special relativity expresses energy (E) with momentum ($\vec{p}$) and mass (m), c is the constant speed of light.

$$E^2 = \vec{p}^2c^2 + m^2c^4$$

Considering that mass is not constant and is a 3 dimensional vector in accordance with the 3 generations of mass of particles. The vector $\vec{m}$ can be defined by (m_1c,m_2c,m_3c) then

$$E^2 = \vec{p}^2c^2 + \vec{m}^2c^2$$

In the Klein-Gordon equation

$$\left[-\bar{h}^2 \frac{\partial^2}{\partial^2 t} \right] \psi(\vec{x},t) = \left[-\bar{h}^2c^2 \frac{\partial^2}{\partial^2 \vec{x}} \right] \psi(\vec{x},t) + m^2c^4 \psi(\vec{x},t)$$

the mass (m) can be replaced by the vector ($\vec{m}$) and the ψ function by $\varphi(\vec{x},\vec{m},t) = \psi(\vec{x},t)\, e^{i\vec{m}c/\bar{h}}$

$$\left[-\bar{h}^2 \quad \frac{\partial^2}{\partial^2 t} \right] \varphi(\vec{x},\vec{m},t) = \left[-\bar{h}^2 c^2 \quad \frac{\partial^2}{\partial^2 \vec{x}} \right] \varphi(\vec{x},\vec{m},t) + \left[-\bar{h}^2 c^2 \quad \frac{\partial^2}{\partial^2 \vec{m}} \right] \varphi(\vec{x},\vec{m},t)$$

where $\bar{h} = h/\pi$ is the reduced Planck constant and *i* is the common imaginary number ($i^2 = -1$),

Dirac equation

The Dirac equation for a free particle is, adding to k = 1,2,3,

$$\left[i\bar{h} \quad \frac{\partial}{\partial t} \right] \psi(\vec{x},t) = \left[mc^2 \alpha_0 - i\bar{h}c \alpha_k \quad \frac{\partial}{\partial x_k} \right] \psi(\vec{x},t)$$

where $\psi(\vec{x},t) = \psi(x_1,x_2,x_3,t)$ is a four dimensional complex vector as solution, called a Dirac spinor.

The equation above can be replaced by a more general equation, adding to k = 1,2,3,

$$\frac{\partial}{\partial t} \varphi(\vec{x},\vec{m},t) = -c \left[\alpha_k \quad \frac{\partial}{\partial x_k} + \beta_k \quad \frac{\partial}{\partial y_k} \right] \varphi(\vec{x},\vec{m},t)$$

where

$$m \alpha_0 = m_1 \beta_1 + m_2 \beta_2 + m_3 \beta_3$$

$$m^2 = m_1^2 + m_2^2 + m_3^2$$

$$\varphi = \psi e^{ix_0 mc/\bar{h}} = \psi e^{i(y_1 m_1 + y_2 m_2 + y_3 m_3)c/\bar{h}}$$

The α_k and β_k shall be chosen to respect the Klein-Gordon equation which means that the following rules shall be respected

$$\alpha_j \beta_k + \beta_k \alpha_j = 0 \qquad \alpha_j^2 = 1 \qquad \beta_k^2 = 1 \qquad (j,k = 1,2,3)$$

Octonion

The octonion ($o \in \mathbb{O}$) is an extension of the quaternion as a solution of the Dirac equation. An octonion is built from two quaternions.

$$o = q_x + q_y\, i_4 \qquad (2)$$

where

$$i_4^2 = -1 \qquad i_4\, i_k = -i_k\, i_4 \qquad (i_j\, i_k)\, i_4 = -i_j\, (i_k\, i_4) \qquad j,k = 1,2,3$$

The quaternions in (2) are orthogonal in the octonion. The basic structure of the quantum wave (1), including its opposite, can be expressed by an octonion.

Lie group

The automorphism group of the octonions is isomorphic to the exceptional Lie group G2 that contains the SU(3) Lie group. All Lie groups used in quantum physics can be derived from the octonions :

$$\mathrm{Auth}(\mathbb{O}) \supset SU(3) \supset SU(2) \supset U(1)$$

Mass is space

In an highly speculative manner and with a lot of shortcuts, some conclusions can be assumed.

- The basic structure is an hypersphere of octonions.
- The basic structure can not be a deeper structure like a sedenion because a deeper structure introduces zero divisors.
- The mass of the particle is the mirror of the momentum on its three dimensions, that's why mass has three generations.
- The mass and the momentum are the same structure and are undistinguishable. They can be exchanged, changing the chirality.
- Mass, as well as energy, are space.

Interaction

An interaction can be summerized by a merge or break in the hypersphere structure.

A particle is a discrete volume in the hypersphere, which contains space and mass. The number of 3D spaces is the number of dimensions and it could be odd or even, which could correspond to fermion or boson.

An interaction is new subdivision of the hypersphere. The new subdivision is a multiplication or a division and is punctual in the 3D space. The reason of the subdivision is still unknown but is relative to the projection into a common 3D space. The punctual interactions form the common 3D space.

The subdivision has two possibilities due to non-commutativity of the octonionic multiplication or division, which introduces random choice or chirality. The non-associativity of the octonion can be seen as a temporal order.

Propagation of the particle (p) is the interaction of the particle by itself

$$p\,\bar{p} = (y_1^2 + y_2^2 + y_3^2) + x_1^2 + x_2^2 + x_3^2 = s^2 + x_1^2 + x_2^2 + x_3^2 = ct^2$$

which corresponds to the Minkowski space.

All the interaction forces are in the mass (potential) dimension. They influence the probability of interaction but not the interaction itself, which remains orthogonal and euclidian. That's why our 3D space is flat.

Entanglement has an obvious solution here. The entangled particles share the same 3D space during and after the interaction. They propagate in their common 3D space. During the interaction of one of the two particles with a third one, the 3D space of the two particles combines randomly with the 3D space of the third one. The set of these interaction points forms our common interaction 3D space.

We can imagine different common interaction spaces, a bit like multiple universes. But only the common interaction spaces with a physical significance has a meaning, that's why there is perhaps only one common interaction space.

Time

Time is not fundamental. The additional dimension to space is the mass, not the time. The hypersphere structure is static, stationary, without time.

Changes occur in the common interaction space thanks to the interactions. Time is the wave propagation or the result of the self-interaction of a particle, relative to the last interaction point.

The principle of causality, which is commonly related to time, can be explained by the non-associativity of the octonionic multiplication or division.

Conclusion

Based on the hope that Nature is simple, this article introduces a new representation of space-time structure of the universe : an hypersphere structure on a multi-dimensional space. Each dimension is an energy quantum with its opposite forming a three dimensional space, as octonion, covering the whole universe. The structure generates the space and the mass, which are equivalent. Time is the result of interactions which take place in a three dimensional reference space. The implications of this hypothesis are vast and go far beyond this short article. This article focuses on build blocks of the structure but does not explore the rules of interactions, which is perhaps a big work.

Whether the theory is correct or not, it seems increasingly clear that the visible common space-time is not a fundamental structure. Elementary particles are supposed to be punctual but have information (charge, mass), so it's an evidence that there is more dimensions to contain this information, not only in an absctract mathematical space but in reality. Elementary particles are also fields that spread to the whole universe and the fields are also a reality. Our space-time is only the visible consequence of the interaction between particles. That’s why calculations based only on our visible space-time can become unstable. To explain the universe, the ether is not necessary and perhaps not space-time either.