

Hypersphere space-time model

Abstract

The origin of the three spatial dimensions as well as the one of time can be deduced from fundamental principles (symmetry). The structure resulting from this construction looks like an hypersphere in which each energy particle constitutes a dimension, forming a loop or a string covering the whole universe. This model shall be linked to the existing theories that are consistent with the reality as measured by experiments.

Article

Space

Quantum wave

Nothing (symmetry) generating something (energy) can be expressed by the addition and the multiplication of an energy quantum (a) and its opposite (\bar{a}) :

$$a + \bar{a} = 0 \text{ (symmetry), } a\bar{a} = 1 \text{ (energy)}$$

The solution is, with $i^2 = -1$

$$a = i, \bar{a} = -i$$

The quantum (a) is a complex number ($a = a_1 + a_2i \in \mathbb{C}$, $a_1, a_2 \in \mathbb{R}$, $i^2 = -1$) so it behaves like a wave, more precisely like the $\pi/2$ phase of a virtual (potential) standing wave covering the whole universe. It's the same for the opposite (\bar{a}).

Quaternion

The quantum (a) and its opposite (\bar{a}) form a pair of complex numbers (a, \bar{a}). These two elements on their own dimension are linked together thanks to a new dimension with the external or vectorial product \wedge : $a \wedge \bar{a} = v$. The vector (v) exists in a three dimensional space ($a, \bar{a}, a \wedge \bar{a}$) that can be represented by a quaternion $q \in \mathbb{H}$

$$q = s + (v) = x_0 + x_1i_1 + x_2i_2 + x_3i_3$$

where $x_0, x_1, x_2, x_3 \in \mathbb{R}$, $i_1^2 = i_2^2 = i_3^2 = i_1i_2i_3 = -1$, $s = x_0 = 0$ is the scalar part, $(v) = (x_1, x_2, x_3)$ is the vectorial part.

Some mathematical operations can be applied to a quaternion (q) or (q') :

closed product: $\mathbf{q}\mathbf{q}' = s s' - \mathbf{v} \cdot \mathbf{v}' + (s\mathbf{v}' + s'\mathbf{v} + \mathbf{v} \wedge \mathbf{v}') \in \mathbb{H}$

conjugate: $\bar{\mathbf{q}} = s - (\mathbf{v}) = x_0 - x_1 i_1 - x_2 i_2 - x_3 i_3$

braket product: $\langle \mathbf{q} | \mathbf{q}' \rangle = \mathbf{q} \bar{\mathbf{q}}'$

euclidian norm: $\|\mathbf{q}\| = \sqrt{\langle \mathbf{q} | \mathbf{q} \rangle} = \sqrt{x_0^2 + x_1^2 + x_2^2 + x_3^2}$

inverse: $\mathbf{q}^{-1} = \bar{\mathbf{q}} / \|\mathbf{q}\|^2 \rightarrow \mathbf{q} \mathbf{q}^{-1} = 1$

right division: $\mathbf{q} / \mathbf{q}' = \mathbf{q} \mathbf{q}'^{-1} = \mathbf{q} \bar{\mathbf{q}}' / \|\mathbf{q}'\|^2 = \langle \mathbf{q} | \mathbf{q}' \rangle / \langle \mathbf{q}' | \mathbf{q}' \rangle$

commutator: $[\mathbf{q}, \mathbf{q}'] = \mathbf{q} \mathbf{q}' - \mathbf{q}' \mathbf{q}$

anticommutator: $\{\mathbf{q}, \mathbf{q}'\} = \mathbf{q} \mathbf{q}' + \mathbf{q}' \mathbf{q}$

scalar product: $\langle \mathbf{q}, \mathbf{q}' \rangle = (\langle \mathbf{q} | \mathbf{q}' \rangle + \langle \mathbf{q}' | \mathbf{q} \rangle) / 2 = s s' + \mathbf{v} \cdot \mathbf{v}' = x_0 x'_0 + x_1 x'_1 + x_2 x'_2 + x_3 x'_3$

unitary quaternion: $\mathbf{u} = \mathbf{q} / \|\mathbf{q}\| \rightarrow \|\mathbf{u}\| = 1$

where $\mathbf{v} \cdot \mathbf{v}' = \mathbf{v}' \cdot \mathbf{v} = x_1 x'_1 + x_2 x'_2 + x_3 x'_3$ is the internal or scalar product of vectors,
 $\mathbf{v} \wedge \mathbf{v}' = \mathbf{v}' \wedge \mathbf{v} = (x_2 x'_3 - x_3 x'_2, x_3 x'_1 - x_1 x'_2, x_1 x'_2 - x_2 x'_1)$ is the external or vectorial product of vectors.

The product is not commutative in general ($\mathbf{q}\mathbf{q}' \neq \mathbf{q}'\mathbf{q}$ if $\mathbf{v} \wedge \mathbf{v}' \neq 0$). That's why the commutator could be non-null and the anticommutator could be null. There is always an inverse for a non-null quaternion because the norm is not null in this case. The braket product acts like a (right) division up to a positive real number, the squared norm $\|\mathbf{q}\|^2$, it is not commutative and even not associative in general.

A quaternion can be expressed by a linear combination of 2×2 complex matrices σ_k

$$\mathbf{q} = x_0 \sigma_0 + x_1 i \sigma_1 + x_2 i \sigma_2 + x_3 i \sigma_3$$

where i is the imaginary number ($i^2 = -1$), σ_0 is the unit matrix and σ_k ($k = 1, 2, 3$) are the Pauli matrices

$$\sigma_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \equiv \mathbb{1} \quad i \sigma_1 = i \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \equiv i_1 \quad i \sigma_2 = i \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \equiv i_2$$

$$i \sigma_3 = i \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \equiv i_3$$

The quaternion is a piece of momentum, a piece of energy. Energy conservation implies that the result of the operation on two pairs of quanta shall not be null and can be reversed, which is in adequation with the quaternionic multiplication and division. So each fundamental element covering the whole universe can be represented by a quaternion. It's the Hamilton's dream.

Homogeneity

Considering that the universe is made of such fundamental energy elements, energy conservation implies a constant (finite) number of such elements or else an homogeneity that means that all elements are identical or even both, constant number and homogeneity. Homogeneity seems to be a principle and it will be supposed.

According to the homogeneity principle, all quaternions can be considered as unitary quaternion ($u \in \mathbb{H}$, $\|u\| = 1$). The easiest way to explain homogeneity is to have only one element that interacts with itself in several ways but this extreme hypothesis will require further study.

Orthogonality

Each fundamental energy element has - or "is" because there is no other characteristic - its own three dimensional (3D) space, perfectly in accordance with the special relativity. Assuming that each element has its own identity, elements are independant to each others and there is an orthogonal representation of them. Each element is on its own dimension, forming an hypersphere of 3D spaces, an hypersphere of quaternions.

Orthogonality between two quaternions is generally defined by a null scalar product between them ($\langle q, q' \rangle = 0$). For pure vectorial quaternions ($q = v$), the orthogonality is defined by a null scalar product of vectors ($v \cdot v' = 0$), which is equivalent to a null anticommutator.

$$\langle v, v' \rangle = v \cdot v' = 0 \quad \Leftrightarrow \quad \{v, v'\} = v \cdot v' + v' \cdot v + (v \wedge v' + v' \wedge v) = 2v \cdot v' = 0$$

Mass

In a first approach, mass existence could be linked to a non negative value of the scalar part ($s = x_0 \neq 0$) of the quaternion, the norm forming the Minkowski space $t^2 = s^2 + x_1^2 + x_2^2 + x_3^2$. But the quaternion is here unitary and purely vectorial ($s = 0$). Furthermore, this approach doesn't answer to a lot of questions.

Mass is specific to some particles, to some set of energy, not to all kind of energy. Mass is expressed inside the Dirac equation. Mass comes from external interaction with the BEH (Higgs) boson, this interaction changes the chirality of the particle. Mass is constant at rest and is spread in 3 generations, according to some relations (e.g. CKM and PMNS matrices). Mass is annihilated by the antiparticle. All these different things shall match. The following explores some research areas but it needs to be developed deeper and more rigorously.

Square root

Any quaternion has a square root. The square root of a pure vectorial unitary quaternion ($u = v$, $\|u\| = 1$, $u^2 = -1$) is

$$\sqrt{u} = \pm \frac{1}{\sqrt{2}} (1 + u)$$

because

$$(1 + u)^2 = 1 + 2u + u^2 = 1 + 2u - 1 = 2u$$

Dirac equation

The Dirac equation for a free particle is, adding to $k = 1,2,3$,

$$\left[i \hbar \frac{\partial}{\partial t} \right] \psi = \left[mc^2 \alpha_0 - i \hbar c \alpha_k \frac{\partial}{\partial x_k} \right] \psi$$

where c is the constant speed of light, $\hbar = h/\pi$ is the reduced Planck constant, i is the imaginary number ($i^2 = -1$), ψ is a four dimensional complex vector as solution.

The goal of the Dirac equation is to obtain the Klein-Gordon equation when its terms are squared. To do this, the α_k ($k = 0,1,2,3$) terms must obey the following rules :

$$\alpha_k^2 = 1 \quad \{\alpha_k, \alpha_{k'}\} = \alpha_k \alpha_{k'} + \alpha_{k'} \alpha_k = 0 \quad (k \neq k')$$

One common representation of α_0 and α_k ($k = 1,2,3$) among many others is the following 4×4 complex matrices where σ_k are the Pauli matrices.

$$\alpha_0 = \begin{bmatrix} \sigma_0 & 0 \\ 0 & -\sigma_0 \end{bmatrix} = \begin{bmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{bmatrix} \quad \alpha_k = \begin{bmatrix} 0 & \sigma_k \\ \sigma_k & 0 \end{bmatrix}$$

By setting $\varphi = \psi e^{ix_0 mc/\hbar}$, $\partial_t = \partial/\partial t$, $\partial_k = \partial/\partial x_k$, the Dirac equation can be rewritten in a simpler and more homogeneous form, adding to $k = 0,1,2,3$

$$\partial_t \varphi + c \alpha_k \partial_k \varphi = 0$$

because

$$\partial_0 \varphi = \psi \partial_0 e^{ix_0 mc/\hbar} = (i mc/\hbar) \psi e^{ix_0 mc/\hbar} = (imc/\hbar) \varphi$$

There are many possible representations of the α_k matrices. The α_0 matrix can be generalized by any $\beta_{k'}$ ($k' = 0,1,2,3$)

$$\beta_{k'} = \begin{bmatrix} \sigma'_{k'} & 0 \\ 0 & -\sigma'_{k'} \end{bmatrix}$$

if $\sigma_k \sigma'_{k'} = \sigma'_{k'} \sigma_k$ because

$$\alpha_k \beta_{k'} + \beta_{k'} \alpha_k = \begin{bmatrix} 0 & -\sigma_k \sigma'_{k'} \\ \sigma_k \sigma'_{k'} & 0 \end{bmatrix} + \begin{bmatrix} 0 & \sigma'_{k'} \sigma_k \\ -\sigma'_{k'} \sigma_k & 0 \end{bmatrix} = 0$$

The $\sigma'_{k'}$ matrix is orthogonal with any σ_k matrix, that can be expressed by independant quaternionic numbers j_k ($k, k'=1,2,3$)

$$j_1^2 = j_2^2 = j_3^2 = j_1 j_2 j_3 = -1 \quad i_k j_{k'} = j_{k'} i_k \quad i \sigma_k \equiv i_k \quad i \sigma'_{k'} \equiv j_{k'}$$

The α_0 matrix can be replaced by a linear combination $y_k \in \mathbb{R}$ of β_k matrices such that

$$\alpha_0^2 = (y_1 \beta_1 + y_2 \beta_2 + y_3 \beta_3)^2 = y_1^2 \beta_1^2 + y_2^2 \beta_2^2 + y_3^2 \beta_3^2 = y_1^2 + y_2^2 + y_3^2 = 1$$

The α_0 matrix represents a (unit) vectorial quaternion, as the α_k ($k = 1,2,3$) do. The Dirac equation becomes, adding to $k = 1,2,3$

$$\partial_t \varphi + c \alpha_k \partial_k \varphi + c \beta_k \partial_0 \varphi = 0$$

By setting $\alpha = (c \sum \alpha_k \partial_k)$, $\beta = (c \sum \beta_k \partial_0)$ and by splitting φ in two parts (φ_L , φ_R) which are quaternions, we get from the Dirac equation

$$\partial_t \varphi_L + \alpha \varphi_R + \beta \varphi_L = 0$$

$$\partial_t \varphi_R + \alpha \varphi_L - \beta \varphi_R = 0$$

By multiplying the second equation by the imaginary i and adding it to the first one, we get, by redefining $\varphi = \varphi_L + i \varphi_R$ and $\bar{\varphi} = \varphi_L - i \varphi_R$

$$\partial_t (\varphi_L + i \varphi_R) + \beta (\varphi_L - i \varphi_R) + i \alpha (\varphi_L - i \varphi_R) = 0$$

$$\partial_t \varphi + \beta \bar{\varphi} + i \alpha \bar{\varphi} = 0$$

$$\partial_t \varphi + (\beta + i \alpha) \bar{\varphi} = 0$$

If φ is a biquaternion, σ is a real vector and $\sigma^2 = 1$, φ_L and φ_R can be defined as

$$\varphi_L = \frac{1}{2} (1 + \sigma) \varphi \quad \varphi_R = \frac{1}{2} (1 - \sigma) \varphi$$

$$\text{SOLUTION ?} \quad D = \varphi_L + \varphi_R i_k \quad \varphi_L = \bar{\varphi}_R$$

then

$$\varphi_L + \varphi_R = \varphi$$

$$\sigma \varphi_L = \varphi_L$$

$$\sigma \varphi_R = -\varphi_R$$

$$\partial_t (\varphi_L + \varphi_R) + c \partial_0 (\varphi_L - \varphi_R) + c \sigma_k \partial_k (\varphi_L + \varphi_R) = (\partial_t + c \sigma \partial_0 + c \sigma_k \partial_k) \varphi = 0$$

φ is a solution of the Dirac equation. As seen in the previous section, φ_L and φ_R have no inverse.

In an highly speculative manner, some conclusions can be assumed.

- A particle is a spinor which is a biquaternion without inverse, that can explain why a particle exists and cannot be divided.
- The mass of the particle is the mirror of the momentum on its three dimensions, that's why mass has three generations.
- The mass and the momentum are undistinguishable and can be exchanged, changing chirality.
- Two spinors are generated, one for the particle and the other one for the antiparticle.

Supposing that $\alpha = \sum \alpha_k$ and $\beta = \sum \beta_k$, $\alpha\beta + \beta\alpha = 0$ and $\alpha\beta = \beta\alpha \rightarrow \alpha\beta = 0 \rightarrow \alpha = (1+\sigma)q$,
 $\beta = q(1-\sigma)$

$$\sigma\sigma' = \sigma'\sigma \Leftrightarrow \sigma' = \lambda\sigma$$

Masse constante, pas de degré de liberté, $y_k = x_k$.

alternate mass-momentum (higgs) ? alternate with itself ?

4 possibilités pour racine carré : ++, --, i×-i; -i×i

The replacement of all α_k by $-\alpha_k$ keeps the α_k relations valid.

Lie group

The set of unitary quaternions (u) is isomorphic to the SU(2) Lie group, the $i\sigma_k$ are the generators. It means that the exponential of a linear combination of the Pauli matrices ($\sigma = x_1\sigma_1 + x_2\sigma_2 + x_3\sigma_3$) is a unitary quaternion. Indeed, the series expansion of the exponential function applied to matrices allows to write

$$u = e^{i\sigma} = e^{i\theta u} = \cos(\theta) + i \sin(\theta) u$$

where $\theta^2 = \|\sigma\|^2 = x_1^2 + x_2^2 + x_3^2$, $u = \sigma/\theta$ is the unitary vector $u^2 = 1$.

The SU(2) Lie group is homomorphic to the SO(3) Lie group. An uncommon representation of the SO(3) generators is

$$J_1 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad J_2 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad J_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

and SO(3) generators with imaginary number i , keeping determinant value as 1, can be represented by

$$I_1 = \begin{bmatrix} 0 & i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad I_2 = \begin{bmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{bmatrix} \quad I_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix}$$

The I_k and J_k matrices, in addition to the two following independant vectorial matrices

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

is similar to the Dirac equations and to a biquaternion (8 dimensions). The two above matrices with the I_k and J_k ones, multiplied by i , also represent the Gell-Mann matrices, the generators of

the SU(3) Lie group. So there is a link between a biquaternion and the SU(3) Lie group, used to represent the strong interaction and the CKM/PMNS matrices.

Interaction

An interaction can be summarized by a merge or break in the (bi)quaternion hypersphere structure.

A particle is a discrete volume of 3D spaces or momentum. The number of 3D spaces is the number of dimensions and it could be odd (biquaternion or fermion) or even (vectorial quaternion or boson).

An interaction is a division of a particle (p) by another particle (p') thanks to a bra-ket product $\langle p|p' \rangle = p\bar{p}'$; But the operation is an equality that can arise in both directions : p and p' generate $p\bar{p}'$ or $p'\bar{p}$ generates p and p'.

The 3D space of $p\bar{p}'$ is a combination of the 3D spaces of p and p', which can be done in two ways (chirality). The 3D space combination is the result of the closed product of (bi)quaternion. The two possible combinations is the result of the non-commutativity of the multiplication of (bi)quaternions. Because of orthogonality of particles p and p', the result $p\bar{p}'$ of the division is perpendicular to the original particles, with two possible directions. That explains why an interaction is punctual and has several possibilities. The result of the interaction generates always one or two particles that share the same 3D space until the next interaction.

An interaction is a break or a merge of 3D spaces. It can also be seen as a rotation in the hypersphere of (bi)quaternions. As seen at the beginning of the article, a quantum of space is like a (bi)quaternionic potential wave and the interaction has effect on the wave. A merge of 3D spaces is a merge or multiplication of the corresponding waves which implies addition of the frequencies. The rotation in the hypersphere acts like a phase change of the wave but the interaction change the reference of the phase, not the phase itself. The interaction is also called collapse of the wave function. The probability of interaction is relative to the sum of the wave functions in the corresponding 3D space reference but the cause of the interaction remains unknown.

Propagation of the particle (p) is the interaction of the particle by itself $\langle p|p \rangle$ that is expresses by the norm or the Minkowski space $\langle p|p \rangle = s^2 + x_1^2 + x_2^2 + x_3^2 = c^2 t^2$.

The standard model $U1 \times SU2 \times SU3$ can be generated by biquaternions because $\mathbb{B} \equiv SU3 \supset SU2 \supset U1$. All the interaction forces are in the mass (potential) dimension. They influence the probability of interaction but not the interaction itself, which remains orthogonal and euclidian. That's why our 3D space is flat.

Entanglement has an obvious solution here. The entangled particles share the same 3D space during and after the interaction. They propagate in their common 3D space. During the interaction of one of the two particles with a third one, the 3D space of the two particles combines randomly with the 3D space of the third one. The set of these interaction points forms our common interaction 3D space.

We can imagine different common interaction spaces, a bit like multiple universes. But only the common interaction spaces with a physical significance has a meaning, that's why there is perhaps only one common interaction space.

Time

Time is not fundamental. The additional dimension to space is the mass, not the time. The hypersphere structure is static, stationary, without time.

Changes occur in the common interaction space thanks to the interactions. Time is the wave propagation or the result of the self-interaction of a particle, relative to the last interaction point.

The principle of causality, which is commonly related to time, can be explained by the non-associativity of the (bi)quaternionic division or bra-ket product. The order of divisions, which means the order of interactions, must generally be taken into account.

Conclusion

Based on the hope that Nature is simple, this article introduces a new representation of space-time structure of the universe : an hypersphere structure on a multi-dimensional space. Each dimension is an energy quantum with its opposite forming a three dimensional space, as biquaternion, covering the whole universe. The structure generates the space and the mass, which are equivalent. Time is the result of interactions which take place in a three dimensional reference space. The implications of this hypothesis are vast and go far beyond this short article.

There is still a long way to involve the whole physic in one theory but this bottom-up approach, from simple principles to more complex structures, in adequation with the current theories which are corroborated by the observed reality, is probably a good way to elaborate a simple and comprehensive theory. This intuitive approach tries to answer to a fundamental question : why has the universe an apparent three dimensional structure in addition of time, which is far from an evidence ?

Whether the theory is correct or not, it seems increasingly clear that the visible common space-time is not a fundamental structure, it's the consequence of the interaction between particles. That's why calculations based only on our visible space-time can become unstable. To explain the universe, the ether is not necessary and perhaps not space-time either.