

# Hypersphere space-time model

## **Abstract**

The origin of the three spatial dimensions as well as that of time is deduced from fundamental principles (symmetry). The structure resulting from this construction looks like an hypersphere of which each energy particle constitutes a dimension, forming a loop or a string covering the whole universe. This model shall be linked to the existing theories that are in adequation with the experience.

## Article

### Space

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Nothing (symmetry) generating something (energy) can be expressed by the addition and the multiplication of an energy quantum ( $a$ ) and its opposite ( $\bar{a}$ ) :

$$a + \bar{a} = 0 \text{ (symmetry), } a\bar{a} = 1 \text{ (energy)} \quad \rightarrow \quad a = i \text{ and } \bar{a} = -i \text{ where } i^2 = -1.$$

The quantum ( $a$ ) is a complex number ( $a = a_1 + a_2 i \in \mathbb{C}$ ,  $a_1, a_2 \in \mathbb{R}$ ,  $i^2 = -1$ ) so it behaves like a wave, more precisely like the  $\pi/2$  phase of a virtual (potential) standing wave covering the whole universe. It's the same for the opposite ( $\bar{a}$ ).

The quantum ( $a$ ) and its opposite ( $\bar{a}$ ) form a pair of complex numbers ( $a, \bar{a}$ ). These two elements on their own dimension are linked together thanks to a new dimension with the external or vectorial product  $\wedge$  :  $a \wedge \bar{a} = v$ . The vector ( $v$ ) exists in a three dimensional space ( $a, \bar{a}, a \wedge \bar{a}$ ) that can be represented by a quaternion  $q \in \mathbb{H}$

$$q = s + (v) = x_0 + x_1 i_1 + x_2 i_2 + x_3 i_3$$

where  $x_0, x_1, x_2, x_3 \in \mathbb{R}$ ,  $i_1^2 = i_2^2 = i_3^2 = i_1 i_2 i_3 = -1$ ,  $x_0 = s = 0$ ,  $(v) = (x_0, x_1, x_3)$ .

A quaternion ( $q$ ) has some properties.

<i>closed product</i> :	$q \in \mathbb{H}, q' \in \mathbb{H} \rightarrow qq' \in \mathbb{H}$
<i>conjugate</i> $\bar{q}$ :	$\bar{q} = s - (v) = x_0 - x_1 i_1 - x_2 i_2 - x_3 i_3$
<i>braket product</i> :	$\langle q q' \rangle = q\bar{q}'$
<i>euclidian norm</i> $\ q\ $ :	$\ q\ ^2 = \langle q q \rangle = x_0^2 + x_1^2 + x_2^2 + x_3^2$
<i>inverse</i> $q^{-1}$ :	$q^{-1} = \bar{q}/\ q\ ^2 \rightarrow qq^{-1} = 1$
<i>right division</i> :	$q/q' = qq'^{-1} = q\bar{q}'/\ q'\ ^2 = \langle q q' \rangle / \langle q' q' \rangle$
<i>commutator</i> :	$[q, q'] = qq' - q'q$
<i>anticommutator</i> :	$\{q, q'\} = qq' + q'q$
<i>unit quaternion</i> $u$ :	$\ u\  = q/\ q\  = 1$

The product is not commutative in general ( $qq' \neq q'q$ ). That's why the commutator could be non-null and the anticommutator could be null. There is always an inverse for a non-null quaternion ( $q \neq 0$ ). The braket product acts like a (right) division up to a real number, the squared norm  $\|q\|^2$ , it is not commutative and even not associative in general.

The quaternion is a piece of momentum, a piece of energy. Energy conservation implies that the result of the operation on two pairs of quanta shall not be null and can be reversed, which is in adequation with the quaternionic multiplication and division. So a fundamental element can be represented by a quaternion. It's the Hamilton's dream.

Considering that the universe is made of such fundamental energy elements, energy conservation implies a constant (finite) number of such elements or else an homogeneity that means that all elements are identical or even both, constant number and homogeneity. Homogeneity seems to be a principle and it will be supposed. The easiest way to explain homogeneity is to have only one element that interacts with itself in several ways but this

extreme hypothesis will require further study. According to the homogeneity principle, all quaternions can be considered as unitary quaternion ( $u \in \mathbb{H}$ ,  $\|u\| = 1$ ).

Each fundamental energy element has - or "is" because there is no other characteristic - its own three dimensional (3D) space, perfectly in accordance with the special relativity. Assuming independancy of elements means that there is an orthogonal representation of them. Each element is on its own dimension, forming an hypersphere of 3D spaces, an hypersphere of quaternions.

## Mass

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In a first approach, mass existence could be linked to a non negative value of the scalar part ( $s = x_0$ ) of the quaternion, the norm forming the Minkowski's formula  $t^2 = s^2 + x_1^2 + x_2^2 + x_3^2$ . But the quaternion is unitary and purely vectorial ( $s = 0$ ). Furthermore, this approach doesn't answer to a lot of questions.

Mass is specific to some particles, to some set of energy, not to all kind of energy. Mass is expressed inside the Dirac equation. Mass comes from external interaction with the BEH (Higgs) boson, this interaction changes the chirality of the particle. Mass is constant at rest and is spread in 3 generations, according to some relations (e.g. CKM and PMNS matrices). Mass is annihilated by the antiparticle. All these different things shall match. The following explores some research areas but it needs to be developed deeper and more rigorously.

## Biquaternion

A vectorial part ( $v$ ) of a quaternion is imaginary i.e.  $(v)^2 < 0$ . By multiplying the vector by the imaginary number  $i$ , the vector becomes real. Real vectors in addition to a complex scalar can generate a biquaternion ( $b \in \mathbb{B} = \mathbb{C} \times \mathbb{H}$ ) which is the sum of a real quaternion and an imaginary quaternion or, equivalently, a quaternion whose real coefficients are replaced by complex coefficients.

$$b = q_x + q_y i$$

$$b = (x_0 + x_1 i_1 + x_2 i_2 + x_3 i_3) + (y_0 + y_1 i_1 + y_2 i_2 + y_3 i_3) i$$

$$b = (x_0 + y_0 i) + (x_1 + y_1 i) i_1 + (x_2 + y_2 i) i_2 + (x_3 + y_3 i) i_3$$

$$b = c_0 + c_1 i_1 + c_2 i_2 + c_3 i_3$$

where  $q_x, q_y \in \mathbb{H}$ ,  $x_j, y_j \in \mathbb{R}$ ,  $c_j = x_j + y_j i \in \mathbb{C}$ ,  $i^2 = i_1^2 = i_2^2 = i_3^2 = i_1 i_2 i_3 = -1$ ,  $i i_j = i_j i$ .

As the quaternion, the product of two biquaternion is a biquaternion.

One definition of the conjugate ( $\bar{b}$ ) of ( $b$ ) is similar to the one of the quaternion

$$\bar{b} = c_0 - c_1 i_1 - c_2 i_2 - c_3 i_3$$

The product of ( $b$ ) with its conjugate ( $\bar{b}$ ) is a complex number.

$$b \bar{b} = c_0^2 + c_1^2 + c_2^2 + c_3^2$$

The biquaternion has a norm  $\|b\|^2 = \|q_x\|^2 + \|q_y\|^2$  which is euclidian.

The inverse of a biquaternion ( $b$ ) is  $b^{-1} = \bar{b}/(b\bar{b})$ . Unlike the quaternion, a biquaternion ( $b$ ) has not always an inverse because  $b\bar{b}$  could be null. A biquaternion ( $b$ ) has a zero-divisor ( $\bar{b}$ ) when [1]

$$b = (1+v) b'$$

where  $b' \in \mathbb{B}$  and  $v = \sum y_j j$  ( $j=1,2,3$ ) is a real vector as  $v^2 = \sum y_j^2 = 1$ . The conjugate of  $b = b_1 b_2$  is  $\bar{b} = \bar{b}_2 \bar{b}_1$  then

$$b\bar{b} = (1+v) b' \bar{b}' (1-v) = b' \bar{b}' (1^2 - v^2) = b' \bar{b}' (1-1) = 0.$$

For ease of representation, a biquaternion is sometimes expressed by a complex linear combination of the Pauli matrices  $\sigma_j$  ( $j=1,2,3$ ) in addition to the unit matrix  $\sigma_0$

$$b = c_0 \sigma_0 + c_1 \sigma_1 + c_2 \sigma_2 + c_3 \sigma_3$$

where  $c_j \in \mathbb{C}$  and

$$\sigma_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \equiv \mathbb{1} \quad \sigma_1 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \equiv i_1 j \quad \sigma_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \equiv i_2 j \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \equiv i_3 j$$

Any  $2 \times 2$  complex matrix can be expressed thanks to a biquaternion, as for example for the real part (use  $i\sigma_0, \sigma_1, i\sigma_2, i\sigma_3$  for the imaginary part)

$$\begin{aligned} (\sigma_0 + \sigma_3)/2 &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} & (\sigma_0 - \sigma_3)/2 &= \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} & (\sigma_2 + i\sigma_1)/2 &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \\ (\sigma_2 - i\sigma_1)/2 &= \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \end{aligned}$$

The characteristics of the biquaternion make it a fundamental structure in several areas.

### **Dirac equation**

The Dirac equation for a free particle is, adding to  $k=1,2,3$

$$\left[ i \hbar \frac{\partial}{\partial t} \right] \psi = \left[ mc^2 \alpha_0 - i \hbar c \alpha_k \frac{\partial}{\partial x_k} \right] \psi$$

where the solution  $\psi$  is a four dimensional complex vector and  $\alpha_k$  ( $k=0,1,2,3$ ) are such that

$$\alpha_k^2 = 1 \quad \{\alpha_k, \alpha_{k'}\} = \alpha_k \alpha_{k'} + \alpha_{k'} \alpha_k = 0 \quad (k \neq k')$$

The following  $4 \times 4$  complex matrices conform to the above rules were  $\sigma_k$  is a Pauli matrix.

$$\alpha_0 = \begin{bmatrix} 0 & -\mathbb{1}i \\ \mathbb{1}i & 0 \end{bmatrix} = \begin{bmatrix} 0 & -i\sigma_0 \\ i\sigma_0 & 0 \end{bmatrix} \quad \alpha_1 = \begin{bmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{bmatrix} \quad \alpha_2 = \begin{bmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{bmatrix}$$

$$\alpha_3 = \begin{bmatrix} 0 & \sigma_3 \\ \sigma_3 & 0 \end{bmatrix}$$

By setting  $\varphi = \psi e^{ix_0 mc/\hbar}$ ,  $\partial_t = \partial/\partial t$ ,  $\partial_k = \partial/\partial x_k$ , the Dirac equation becomes

$$i \hbar (\partial_t + c \alpha_k \partial_k) \varphi = 0$$

or in two equations ( $c = 1$ )

$$\partial_t \varphi_L + (\sigma_k \partial_k - i \sigma_0 \partial_0) \varphi_R = 0$$

$$\partial_t \varphi_R + (\sigma_k \partial_k + i \sigma_0 \partial_0) \varphi_L = 0$$

If  $\varphi$  is a biquaternion,  $\sigma$  is a real vector and  $\sigma^2 = 1$ ,  $\varphi_L$  and  $\varphi_R$  can be defined as

$$\varphi_L = \frac{1}{2} (1+\sigma)\varphi \quad \varphi_R = \frac{1}{2} (1-\sigma)\varphi$$

then

$$\varphi_L + \varphi_R = \varphi$$

$$\sigma \varphi_L = \varphi_L$$

$$\sigma \varphi_R = -\varphi_R$$

$$\partial_t(\varphi_L + \varphi_R) + c \partial_0(\varphi_L - \varphi_R) + c \sigma_k \partial_k(\varphi_L + \varphi_R) = (\partial_t + c \sigma \partial_0 + c \sigma_k \partial_k) \varphi = 0$$

$\varphi$  is a solution of the Dirac equation. As seen in the previous section,  $\varphi_L$  and  $\varphi_R$  have no inverse.

In an highly speculative manner, some conclusions can be assumed.

- A particle is a spinor which is an hyperquaternion without inverse, that can explain why a particle exists and cannot be divided.
- The mass of the particle is the mirror of the momentum on its three dimensions, that's why mass has three generations.
- The mass and the momentum are undistinguishable and can be exchanged, changing chirality.
- Two spinors are generated, one for the particle and the other one for the antiparticle.

alternate mass-momentum (higgs) ? alternate with itself ?

**Lie group**

The set of unitary quaternions ( $u$ ) is isomorphic to the  $SU(2)$  Lie group, the  $i\sigma_k$  are the generators. It means that the exponential of a linear combination of the Pauli matrices ( $\sigma = x_1\sigma_1 + x_2\sigma_2 + x_3\sigma_3$ ) is a unitary quaternion. Indeed, the series expansion of the exponential function applied to matrices allows to write

$$u = e^{i\sigma} = e^{i\theta u} = \cos(\theta) + i \sin(\theta) u$$

where  $\theta^2 = \|\sigma\|^2 = x_1^2 + x_2^2 + x_3^2$ ,  $u = \sigma/\theta$  is the unitary vector  $u^2 = 1$ .

The  $SU(2)$  Lie group is homomorphic to the  $SO(3)$  Lie group. An uncommon representation of the  $SO(3)$  generators is

$$J_1 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad J_2 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad J_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

and  $iSO(3)$  generators can be defined by

$$I_1 = \begin{bmatrix} 0 & i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad I_2 = \begin{bmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{bmatrix} \quad I_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix}$$

The determinant of  $I_k$  is 1.

The multiplication of the matrices as well as the unit matrix by  $i$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

two dimensions similar to real-imaginary dimension ?

gives the Gell-Mann matrices, they are generators of the  $SU(3)$  Lie group. So there is a link between an hyperquaternion and the  $SU(3)$  Lie group.

what about product = 0 ?

## Interaction

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Everything is phase change.

interaction is punctual

A particle is a discrete volume of momentum. The number of momentum is the number of dimensions and it could be odd (biquaternion or fermion) or even (quaternion or boson).

An interaction is a division of a particle ( $p$ ) by another particle ( $p'$ ) thanks to a bra-ket product  $\langle p|p' \rangle = p\bar{p}$ ; Division arises in a reference space between the two particles, called the common

interaction space. The particles are perpendicular, so the result of the division is perpendicular with two possibilities, depending of the order of division.

Propagation of the particle (p) is the interaction of the particle by itself  $\langle p|p\rangle$  that expresses the norm or the Poincaré-Minkowski's formula  $\langle p|\bar{p}\rangle = s^2+x_1^2+x_2^2+x_3^2 = c^2t^2$ .

An interaction between particles (p) and (p') can create or destroy particles, it can also generate a collision for vectorial particles  $p = v, p' = v'$

$$\langle\langle p|p'|p\rangle = p\bar{p}'\bar{p} = v(-v')(-v) = vv(-v') = -v' \quad \langle p|\langle p'|p\rangle = p\bar{p}'' = vv(-v') = -v'$$

Each particle has its own 3D space but there is apparently a common 3D space. An interaction is a 3D space relation between particles, they share the same 3D space. An interaction is a projection of the 3D space of the particles onto this common interaction space.

An interaction is a break or a division in the common interaction space. The division is a bra-ket product between orthogonal bivectors, which always generates a bivector in the common interaction space.

The standard model  $U1 \times SU2 \times SU3$  can be generated by hyperquaternions. All the interaction forces are in the mass (potential) dimension. They influence the probability of interaction but not the interaction itself.

An interaction is a rotation, the position is the phase and the interaction point is the phase change.

A particle can be seen as a punctual projection of space-mass on the common 3D space.

Energy is the volume of the hypersphere projected onto the common interaction space in a parallel (kinetic) or orthogonal (potential) way.

The common interaction space is euclidian, so flat, because the norm of hyperquaternions is euclidian. The common interaction space remains flat despite the interactions because the forces are only potential.

Entanglement has an obvious solution here. The entangled particles share the same 3D space but outside the common interaction space after interaction. They propagate in their own 3D space but during interaction, their space combines with the common interaction space, which can randomly be done in several different ways.

We can imagine different common interaction spaces, a bit like multiple universes. But there may only be one, the only one where interactions have physical significance.

## Time

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Time is not fundamental. It's the wave propagation or self-interaction of a particle. Time or propagation is relative to the last interaction point.

The principle of causality, which is commonly related to time, can be explained by the non-associativity of the (bi)quaternionic division or bra-ket product. The order of divisions, which means the order of interactions, must generally be taken into account.

## **Conclusion**

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Based on the hope that Nature is simple, this article introduces a new representation of space-time structure of the universe : an hypersphere structure on a multi-dimensional space, each dimension is an energy quantum with its opposite forming a biquaternion covering the whole universe. The implications of this hypothesis are vast and go far beyond this short article.

There is still a long way to involve the whole physic in one theory but this bottom-up approach, from simple principles to more complex structures, in adequation with the observed reality, is probably a good way to elaborate a simple and comprehensive theory. This intuitive approach tries to answer to a fundamental question : why has the universe an apparent three dimensional structure in addition of time, which is far from an evidence ?

Whether the theory is correct or not, it seems increasingly clear that the visible common space-time is not a fundamental structure, it's the consequence of the interaction between particles. That's why calculations based only on our visible space-time can become unstable. To explain the universe, the ether is not necessary and perhaps not space-time either.

## **References**

[1] CASANOVA Gaston, "L'algèbre vectorielle", *Que sais-je?* n°1657 p49 (1976)

Ibidem p37

## Puzzle

- énergie
  - moment = addition ?, énergie = multiplication
  - conservation du moment ( $d/dx=0$ ) -> phase varie mais pas amplitude (norme)
- quaternion
  - déterminant de matrice = volume
  - quotient de vecteurs, bivecteur = matrice de Pauli,
  - matrice orthogonale → conjugué complexe (aussi matrice)
  - bra-ket = division, non associatif, indiscernable
  - moment (vecteur)
  - propagation (Minkowski, unitarité  $A \rightarrow U^{-1}AU$ )
  - $SU2 \rightarrow SU3$  via  $SU2 \times SU2 \times SU2 = U3$  ou  $SO3$
  - $\det(e^H) = e^{\text{Tr}(H)} \rightarrow \text{Tr}(H) = 0$  pour  $\det = 1$
  - $SU(n)$  : matrice complexe unitaire  $\det=1 \rightarrow$  algèbre de Lie matrice anti-hermitienne de trace=0
  - $\det = \text{volume}$
- masse
  - antimatière pour masse négative (équation Dirac  $\gamma_0$ ), symétrie masse/espace car pas d'inverse biquaternion (norme = 0), chiralité compense masse
  - propagation = 0 si masse = vitesse ?
  - temps/masse crée alternance chiralité
  - double couverture  $SU(2)$  de  $SO(3)$  crée masse ?
  - familles : rotation mais énergies différentes aussi
  - biquaternion (s, iv), sans inverse,  $i^4 \rightarrow 1$  (chapeau mexicain)
  - variable en théorie mais constante en pratique(aléatoire ? seule réaction possible ?)
  - brisure de symétrie -> probabilités asymétriques (pas 1/2)
- interaction
  - constante de structure (./ vitesses)
  - interaction ponctuelle → projection
  - matérialisation d'un choix par rapport à une référence
- temps
  - propagation par auto interaction (norme)
  - source de choix possibles
  - NB :
    - se baser sur c et h constants
    - imprécision/indétermination augmente avec le temps
    - exclusion de Pauli agrandit univers
    - projecteur  $E^2=E \rightarrow E=0$  ou 1
    - double couverture  $SO3$  par  $SU2 \rightarrow$  spin
    - boson virtuel si distance inférieure à fréquence
    - Lagrangien = onde car échange de valeurs cos/sin
    - où est la propriété de quantité de masse (interaction Higgs) ?
    - gravitation = rotation cône de lumière
    - interaction = brisure symétrie, de phase, d'onde stationnaire
    - intrication = pas orthogonal, dépendance linéaire
    - énergie empruntée de  $x dt$  à cause onde et déphasage
    - const cosmologique : force croit avec distance
    - temps et probabilité liés (improbable si temps court), temps flou, augmente entropie
    - action =  $mL^2/t$
    - $d(\exp(m^2)f(x,t))/dm = 2m(\exp(m^2)f(x,t))$
    - $E^2 = m^2(c^2)^2 + p^2c^2 = (mc)^2c^2 + (mv)^2c^2$