Hypersphere space-time model

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Abstract

The origin of the three spatial dimensions as well as that of time is deduced from fundamental principles (symmetry). The structure resulting from this construction looks like an hypersphere of which each energy particle constitutes a dimension, forming a loop or a string covering the whole universe. This model shall be linked to the existing theories that are in adequation with the experience.

Article

Space

Nothing (symmetry) generating something (energy) can be expressed by the addition and the multiplication of an energy quantum (a) and its opposite ($\bar{a}$) :

$$a + \bar{a} = 0 \text{ (symmetry)}, \ a\bar{a} = 1 \text{ (energy)} \quad \rightarrow a = i \text{ and } \bar{a} = -i \text{ where } i^2 = -1.$$

The quantum (a) is a complex number ($a = a_1+a_2 i \in \mathbb{C}$, a_1, $a_2 \in \mathbb{R}$, $i^2 = -1$) so it behaves like a wave, more precisely like the π/2 phase of a virtual (potential) standing wave covering the whole universe. It's the same for the opposite ($\bar{a}$).

The quantum (a) and its opposite ($\bar{a}$) form a pair of complex numbers ($a,\bar{a}$). These two elements on their own dimension are linked together thanks to a new dimension with the external or vectorial product ∧ : $a \wedge \bar{a} = v$. The vector (v) exists in a three dimensional space ($a,\bar{a},a\wedge\bar{a}$) that can be represented by a quaternion $q \in \mathbb{H}$

$$q = s + (v) = x_0 + x_1 i_1 + x_2 i_2 + x_3 i_3$$

where $x_0, x_1, x_2, x_3 \in \mathbb{R}$, $i_1^2 = i_2^2 = i_3^2 = i_1 i_2 i_3 = -1$, $s = x_0$, $(v) = (x_1,x_2,x_3)$

more precisely by a vectorial quaternion where s = 0. The quaternion is a piece of momentum, a piece of energy. Energy conservation implies that the result of the operation on two pairs of quanta shall not be null and can be reversed, which is in adequation with the quaternionic multiplication and division. So a fundamental element can be represented by a quaternion. It's the Hamilton's dream.

Considering that the universe is made of such fundamental energy elements, energy conservation implies a constant (finite) number of such elements or else an homogeneity that means that all elements are identical or even both, constant number and homogeneity. Homogeneity seems to be a principle and it will be supposed. The easiest way to explain homogeneity is to have only one element that interacts with itself in several ways but this extreme hypothesis will require further study.

According to the homogeneity principle, all quaternions can be considered as unitary quaternion (u) of norm 1 ($||u|| = 1$), following the definition

$$||q||^2 = \langle q|q\rangle = q\bar{q} = x_0^2+x_1^2+x_2^2+x_3^2$$

where $||q||$ is the norm, $\langle q|q'\rangle = q\bar{q}'$ is the bra-ket product and $\bar{q} = x_0 - x_1 i_1 - x_2 i_2 - x_3 i_3$ is the conjugate of (q). Note that $\bar{q}/||q||^2$ is the inverse of any $q \neq 0$ and that the bra-ket product acts like a (right) division, not commutative and even not associative in general.

Each fundamental energy element has - or "is" because there is no other characteristic - its own three dimensional (3D) space, perfectly in accordance with the special relativity. Assuming independancy of elements means that there is an orthogonal representation of them. Each element is on its own dimension, forming an hypersphere of 3D spaces, an hypersphere of quaternions.

Mass

In a first approach, mass existence could be linked to a non negative value of (s) in the quaternion. But the quaternion is unitary and purely vectorial (s = 0). Furthermore, this approach doesn't answer to a lot of questions.

Mass is specific to some particles, to some set of energy, not to all kind of energy. Mass is expressed inside the Dirac equation. Mass comes from external interaction with the BEH (Higgs) boson, this interaction changes the chirality of the particle. Mass is constant at rest and is spread in 3 generations, according to some relations (CKM and PMNS matrices). Mass is annihilated by the antiparticle. All these different things shall match. The following explores some research areas but it needs to be developed deeper and more rigorously.

Hyperquaternion

Let's assume that the element x_k (k = 0,1,2,3) of the quaternion is not a real number but a unitary quaternion u_k, forming an hyperquaternion (r)

$$r = u_0 + u_1 j_1 + u_2 j_2 + u_3 j_3$$

$$\text{where } u_k = x_{k0} + x_{k1} i_1 + x_{k2} i_2 + x_{k3} i_3 \in \mathbb{H}, \quad \Sigma \|x_{kj}\|^2 = 1, \quad j_1^2 = j_2^2 = j_3^2 = j_1 j_2 j_3 = -1, \quad j_k i_l = -i_l j_k$$

The definition of the conjugate ($\overline{r}$) of (r) is similar to the one of the quarternion

$$\overline{r} = \overline{u}_0 - \overline{u}_1 j_1 - \overline{u}_2 j_2 - \overline{u}_3 j_3$$

As the quaternion, the multiplication of two hyperquaternions is an hyperquaternion. The hyperquaternion has a norm $\|r\|^2 = \Sigma \|u_k\|^2$ which is euclidian. Unlike the quaternion, an hyperquaternion (r) has no inverse and has a zero-divisor ($\overline{r}$) when [1]

$$r = r'(1+i_k j_l) \quad \text{where } r' \in \mathbb{H}\times\mathbb{H}$$

because $r = r_1 r_2$ implies $\overline{r} = \overline{r}_2 \overline{r}_1$ and thus

$$r\overline{r} = q(1+i_k j_l)\,(1-i_k j_l)q = q\,(1^2-(i_k j_l)^2)\,\overline{q} = q\,(1-1)\,\overline{q} = 0.$$

Unitary quaternion can be expressed thanks to the SU(2) Lie group generators $i\sigma_k$ where σ_k are the Pauli matrices

$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Note that $\sigma_k^2 = \mathbb{1}$ and $\sigma_k\sigma_{k'} + \sigma_{k'}\sigma_k = \{\sigma_k,\sigma_{k'}\} = 0$ for $k \neq k'$.

A unitary quaternion (u) is the exponential of a linear combination (σ) of the generators such as $\sigma = x_1\sigma_1 + x_2\sigma_2 + x_3\sigma_3$. The series expansion of the exponential function applied to matrices allows to write

$$u = e^{i\sigma} = e^{i\theta\upsilon} = \cos(\theta) + i\,\sin(\theta)\,\upsilon$$

where $\theta = ||\sigma|| = \sqrt{x_1^2 + x_2^2 + x_3^2}$, $\upsilon = \sigma/\theta$ is the unitary vector

In a similar way, an hyperquaternion (r) can be generated by a linear combination of 9 extended Pauli matrices σ_{lk}, k,l = 1,2,3, in addition to the unit matrix $\mathbb{1}$.

$$\sigma_{1k} = \begin{bmatrix} 0 & \sigma_k \\ \sigma_k & 0 \end{bmatrix} \qquad \sigma_{2k} = \begin{bmatrix} 0 & -i\sigma_k \\ i\sigma_k & 0 \end{bmatrix} \qquad \sigma_{3k} = \begin{bmatrix} \sigma_k & 0 \\ 0 & -\sigma_k \end{bmatrix}$$

Like the Pauli matrices, $\sigma_{lk}^2 = \mathbb{1}$ and $\{\sigma_{lk},\sigma_{l'k'}\} = 0$ for $l \neq l'$ and $k = k'$ or $l = l'$ and $k \neq k'$.

Any hyperquaternion can be written as

$$r = x_0\mathbb{1} + \Sigma\, x_{lk}\sigma_{lk}$$

Dirac equation

The Dirac equation is

$$\left[i\hbar \frac{\partial}{\partial t} \right] \psi = \left[mc^2\, \alpha_0 - i\,\hbar c\, \alpha_j \frac{\partial}{\partial x_j} \right] \psi$$

$$\alpha_0 = \begin{bmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{bmatrix} \qquad \alpha_j = \begin{bmatrix} 0 & \sigma_j \\ \sigma_j & 0 \end{bmatrix} \qquad j = 1,2,3$$

By setting $\varphi = \psi\, e^{i(m_1y_1+m_2y_2+m_3y_3)c/\hbar}$ such that $m_1^2 + m_2^2 + m_3^2 = m^2$ and replacing the α_0 matrix by the three matrices

$$\beta_1 = \begin{bmatrix} \sigma_1 & 0 \\ 0 & -\sigma_1 \end{bmatrix} \qquad \beta_2 = \begin{bmatrix} \sigma_2 & 0 \\ 0 & -\sigma_2 \end{bmatrix} \qquad \beta_3 = \begin{bmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{bmatrix}$$

the Dirac equation becomes

$$i\hbar \left[\frac{\partial}{\partial t} + c\, \alpha_j \frac{\partial}{\partial x_j} + c\, \beta_j \frac{\partial}{\partial y_j} \right] \varphi = 0 \qquad j = 1,2,3$$

Any hyperquaternion (φ) can be split in two spinors φ_L and φ_R that are also hyperquaternions

$$\varphi = \varphi_L + \varphi_R = \left[\frac{1}{2}\, \varphi(1+\sigma) \right] + \left[\frac{1}{2}\, \varphi(1-\sigma) \right]$$

where σ is an hyperquaternion and $\sigma^2 = 1$. These spinors have the following characteristics

$$\varphi_L\sigma = \varphi_L \quad \text{and} \quad \varphi_R\sigma = -\varphi_R$$

so they are a solution of the Dirac equation. As seen in the previous section, φ_L and φ_R have no inverse.

In an highly speculative manner, some conclusions can be assumed.

- A particle is a spinor which is an hyperquaternion without inverse, that can explain why a particle exists and cannot be divided.
- The mass of the particle is the mirror of the momentum on its three dimensions, that's why mass has three generations.
- The mass and the momentum are undistinguishable and can be exchanged, changing chirality.
- Two spinors are generated, one for the particle and the other one for the antiparticle.

SU(3) Lie group

The SU(2) Lie group is homomorphic to the SO(3) Lie group. An uncommon representation of the SO(3) generators is

$$J_1 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad J_2 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \qquad J_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

and *i*SO(3) generators can be defined by

$$I_1 = \begin{bmatrix} 0 & i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad I_2 = \begin{bmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{bmatrix} \qquad I_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix}$$

because the determinant is 1 but $I_j^2 = -1$.

The multiplication of the matrices as well as the unit matrix by *i*

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

gives the Gell-Mann matrices, they are generators of the SU(3) Lie group. So there is a link between an hyperquaternion and the SU(3) Lie group. **what about product = 0 ?**

Interaction

Everything is phase change.

A particle is a discrete volume of momentum. The number of momentum is the number of dimensions and it could be odd (hyperquaternion or fermion) or even (quaternion or boson).

An interaction is a division of a particle (p) by another particle (p') thanks to a bra-ket product $<p|p'> = p\overline{p'}$; Division arises in a reference space, called the common interaction space. The particles are perpendicular, so the result of the division is perpendicular with two possibilities, depending of the order of division.

Propagation of the particle (p) is the interaction of the particle by itself $<p|p>$ that expresses the norm or the Poincaré-Minkowski's formula $<p|\overline{p}> = s^2+x_1^2+x_2^2+x_3^2 = c^2t^2$.

An interaction between particles (p) and (p') can create or destroy particles, it can also generate a collision for vectorial particles p = v, p' = v'

$$<<p|p'|p> = p\overline{p'}\overline{p} = v(-v')(-v) = vv(-v') = -v' \qquad <p|<p'|p> = p\overline{p''} = vv(-v') = -v'$$

Each particle has its own 3D space but there is apparently a common 3D space. An interaction is a 3D space relation between particles, they share the same 3D space. An interaction is a projection of the 3D space of the particles onto this common interaction space.

An interaction is a break or a division in the common interaction space. The division is a bra-ket product between orthogonal bivectors, which always generates a bivector in the common interaction space.

The standard model U1×SU2×SU3 can be generated by hyperquaternions. All the interaction forces are in the mass (potential) dimension. They influence the probability of interaction but not the interaction itself.

An interaction is a rotation, the position is the phase and the interaction point is the phase change.

A particle can be seen as a punctual projection of space-mass on the common 3D space.

Energy is the volume of the hypersphere projected onto the common interaction space in a parallel (kinetic) or orthogonal (potential) way.

The common interaction space is euclidian, so flat, because the norm of hyperquaternions is euclidian. The common interaction space remains flat despite the interactions because the forces are only potential.

Entanglement has an obvious solution here. The entangled particles share the same 3D space but outside the common interaction space after interaction. They propagate in their own 3D space but during interaction, their space combines with the common interaction space, which can randomly be done in several different ways.

We can imagine different common interaction spaces, a bit like multiple universes. But there may only be one, the only one where interactions have physical significance.

Time

Time is not fundamental. It's the wave propagation or self-interaction of a particle. Time or propagation is relative to the last interaction point.

The principle of causality, which is commonly related to time, can be explained by the non-associativity of the (bi)quaternionic division or bra-ket product. The order of divisions, which means the order of interactions, must generally be taken into account.

Conclusion

Based on the hope that Nature is simple, this article introduces a new representation of space-time structure of the universe : an hypersphere structure on a multi-dimensional space, each dimension is an energy quantum with its opposite forming a hyperquaternion covering the whole universe. The implications of this hypothesis are vast and go far beyond this short article.

There is still a long way to envolve the whole physic in one theory but this bottom-up approach, from simple principles to more complex structures, in adequation with the observed reality, is probably a good way to elaborate a simple and comprehensive theory. This intuitive approach tries to answer to a fundamental question : why has the universe an apparent three dimensional structure in addition of time, which is far from an evidence ?

Whether the theory is correct or not, it seems increasingly clear that the visible common space-time is not a fundamental structure, it’s the consequence of the interaction between particles. That’s why calculations based only on our visible space-time can become unstable.To explain the universe, the ether is not necessary and perhaps not space-time either.

References

[1] CASANOVA Gaston, "L'algèbre vectorielle", *Que sais-je?* n°1657 p49 (1976)

Ibidem p69

Puzzle

- énergie
  - moment = addition ?, énergie = multiplication
  - conservation du moment (d/dx=0) -> phase varie mais pas amplitude (norme)
- quaternion
  - déterminant de matrice = volume
  - quotient de vecteurs, bivecteur = matrice de Pauli,
  - matrice orthogonale → conjugué complexe (aussi matrice)
  - bra-ket = division, non associatif, indiscernable
  - moment (vecteur)
  - propagation (Minkowski, unitarité A -> $U^{-1}AU$)
  - SU2 -> SU3 via SU2xSU2xSU2=U3 ou SO3
  - $\det(e^H) = e^{Tr(H)}$ → Tr(H) = 0 pour det = 1
  - SU(n) : matrice complexe unitaire det=1 → algèbre de Lie matrice anti-hermitienne de trace=0
  - det = volume
- masse
  - antimatière pour masse négative (équation Dirac γ_0), symétrie masse/espace car pas d'inverse biquaternion (norme = 0), chiralité compense masse
  - propagation = 0 si masse = vitesse ?
  - temps/masse crée alternance chiralité
  - double couverture SU(2) de SO(3) crée masse ?
  - familles : rotation mais énergies différentes aussi
  - biquaternion (s, iv), sans inverse, i4 -> 1 (chapeau mexicain)
  - variable en théorie mais constante en pratique(aléatoire ? seule réaction possible ?)
  - brisure de symétrie -> probabilités asymétriques (pas 1/2)
- interaction
  - constante de structure (./. vitesses)
  - interaction ponctuelle → projection
  - matérialisation d'un choix par rapport à une référence
- temps
  - propagation par auto interaction (norme)
  - source de choix possibles
  - NB :
    - se baser sur c et h constants
    - imprécision/indétermination augmente avec le temps
    - exclusion de Pauli agrandit univers
    - projecteur $E^2=E$ -> E=0 ou 1
    - double converture SO3 par SU2 -> spin
    - boson virtuel si distance inférieure à fréquence
    - Lagrangien = onde car échange de valeurs cos/sin
    - où est la propriété de quantité de masse (interaction Higgs) ?
    - gravitation = rotation cône de lumière
    - interaction = brisure symétrie, de phase, d'onde stationnaire
    - intrication = pas orthogonal, dépendance linéaire
    - énergie empruntée de x dt à cause onde et déphasage
    - const cosmologique : force croit avec distance
    - temps et probabilité liés (improbable si temps court), temps flou, augmente entropie
    - action = mL^2/t
    - $d(\exp(m^2)f(x,t))/dm = 2m(\exp(m^2)f(x,t))$
    - $E^2 = m^2(c^2)^2 + p^2c^2 = (mc)^2c^2 + (mv)^2c^2$