

Hypersphere space-time model

(draft)

Abstract

The origin of the three spatial dimensions as well as that of time is deduced from fundamental principles (symmetry, homogeneity). The structure resulting from this construction looks like an hypersphere of which each energy particle constitutes a dimension, forming a loop or a string covering the whole universe. This model shall be linked to the existing theories that are in adequation with the experience.

Generation of space

Nothing (*symmetry*) generating something (*energy*) can be expressed by the addition and the multiplication of an energy quantum (a) and its opposite (\bar{a}) :

$$a + \bar{a} = 0 \text{ (*symmetry*)}, a\bar{a} = 1 \text{ (*energy*)} \rightarrow a = i \text{ and } \bar{a} = -i \text{ where } i^2 = -1.$$

The quantum (a) is a complex number ($a = a_1 + ia_2 \in \mathbb{C}$, $a_1, a_2 \in \mathbb{R}$, $i^2 = -1$) so it behaves like a wave, more precisely like the $\pi/2$ phase of a virtual (potential) standing wave. It's the same for the opposite (\bar{a}).

The quantum (a) and its opposite (\bar{a}) form a pair of complex numbers (a, \bar{a}), commonly referred as a spinor or as a quaternion. **POURQUOI quaternion, multiplication ? À cause de l'indiscernabilité**

A quaternion (q) is defined by

$$q = a + \bar{a}j = a_1 + ia_2 + \bar{a}_1j + i\bar{a}_2j = a_1 + ia_2 + j\bar{a}_1 + k\bar{a}_2 \quad \text{where } j^2 = -1, ij = k = -ji,$$

or in a more general notation

$$q = s + ix + jy + kz = s + \vec{v} \quad \text{where } s, x, y, z \text{ are scalar } (\in \mathbb{R}), i^2 = j^2 = k^2 = ijk = -1, \\ \vec{v} = (x, y, z) \text{ is a 3 dimensional vector}$$

The result of the product of two non-zero quaternions (q) and (q') is also a non-zero quaternion

$$\begin{aligned} qq' &= (s + \vec{v})(s' + \vec{v}') = (s + ix + jy + kz)(s' + ix' + jy' + kz') \\ &= ss' + isx' + jsy' + ks z' + ixs' - xx' + kxy' - jxz' + jys' - kyx' - yy' + iyz' + kzs' + jzx' - izy' - zz' \\ &= (ss' - xx' - yy' - zz') + s(ix' + jy' + kz') + s'(ix + jy + kz) + (i(yz' - zy') + j(zx' - xz') + k(xy' - x'y)) \\ &= (ss' - \vec{v} \cdot \vec{v}') + (s\vec{v}' + s'\vec{v} + \vec{v} \times \vec{v}')$$

where the scalar part is ($ss' - \vec{v} \cdot \vec{v}'$) and the vectorial part is ($s\vec{v}' + s'\vec{v} + \vec{v} \times \vec{v}'$), $\vec{v} \cdot \vec{v}' = xx' + yy' + zz'$ is the scalar product of vectors and $\vec{v} \times \vec{v}' = (yz' - y'z, zx' - z'x, xy' - x'y)$ the vectorial product of vectors. The product of quaternions is not commutative because the vectorial product is anti-commutative ($\vec{v} \times \vec{v}' = -\vec{v}' \times \vec{v}$).

The **conjugate** (\bar{q}) of the quaternion (q) is defined by

$$\bar{q} = s - \vec{v} = s - ix - jy - kz$$

which implies

the **bra-ket** product $\langle | \rangle$
 $\langle q | q' \rangle = q\bar{q}'$

the **norm** $\|q\|$
 $\|q\|^2 = \langle q | q \rangle = q\bar{q} = s^2 + \vec{v} \cdot \vec{v} = s^2 + x^2 + y^2 + z^2 \in \mathbb{R}$

the **inverse** (q^{-1})
 $q^{-1} = \bar{q}/\|q\|^2 \quad \text{because } qq^{-1} = q\bar{q}/\|q\|^2 = 1$

the right **division**, as the bra-ket product divided by the norm,
 $q/q' = qq'^{-1} = q\bar{q}'/\|q'\|^2 = \langle q | q' \rangle / \langle q' | q \rangle$

orthogonality between (q) and (q') if

$$\|q+q'\|^2 = \|q\|^2 + \|q'\|^2$$

$$\rightarrow \langle q+q' | q+q' \rangle - \langle q | q \rangle - \langle q' | q' \rangle = \langle q | q \rangle + \langle q' | q \rangle + \langle q' | q' \rangle - \langle q | q \rangle - \langle q' | q' \rangle = 0$$

$$\rightarrow \langle q | q' \rangle + \langle q' | q \rangle = 0$$

$$\rightarrow (ss' + \vec{v} \cdot \vec{v}') + (s' \vec{v} - s \vec{v}' - \vec{v} \times \vec{v}') + (s' \vec{v}' + s \vec{v} \cdot \vec{v}') + (s \vec{v}' - s' \vec{v} - \vec{v}' \times \vec{v}) = 0$$

$$\rightarrow ss' + \vec{v} \cdot \vec{v}' = 0$$

Implication de l'orthogonalité, indiscernabilité ?

The quaternion (\tilde{q}) is the reverse of order or inverse of chirality of the quaternion (q) when one coordinate (also the scalar one) is reversed ($ij = -k$).

$$-\bar{q} = -s + ix + jy + kz$$

The bra-ket product is not always commutative, because of the non-commutativity of the multiplication of quaternions, but it is not associative too if $\bar{q}q' \neq q'q$

$$\begin{aligned} \langle q'' | q' \rangle | q \rangle &= \langle q'' \bar{q}' | q \rangle = q'' \bar{q}' \bar{q} \\ \langle q'' | \langle q' | q \rangle \rangle &= \langle q'' | q' \bar{q} \rangle = q'' q \bar{q}' \end{aligned}$$

According to the *homogeneity* principle, all quaternions issued by the (a, \bar{a}) quanta are identicals and are supposed to be unitary ($\|q\| = 1$). Such a quaternion is here called a **quantion**. A quantion has a three dimensional (3D) vectorial part only which is a velocity vector. The quantions are the 3D space but each quantions has its own 3D space referential. That's the generation of 3D space.

By the far from evidence hypothesis that energy is constant, so finite, there is a constant number N of quantions in the universe. If quantions are indistinguishable, they are independant to each others, so orthogonal. Then they form the surface of an hypersphere in a vectorial space of size N , corresponding to the number of quantions.

Each energy quantum wave (a) has the same amplitude and same frequency and cover the whole universe on its dimension, so half of the wavelength of (a) can be considered as the size of the universe.

The total energy of the universe is all the quantions.

Position in space

A set of quantions can be projected on a 3D direction to form a particle. The multiplication of (n) quantions forms a wave with a frequency of (n) : q^n

But there is others solutions if (n) has (m_i) as divider, one solution is the sum of the m_i waves of frequency (n/m_i) : $m_i q^{n/m_i}$

So the sum of all possibilities is the sum of all dividers (m_i) of (n) : $m_i q^{n/m_i}$

It's a Fourier serie where the sum of waves generates a more accurate virtual position in the space.

Generation of time

Everything is static, without change, because of the standing state. Time is generated from change and from causality, that orders the change.

Propagation

Quantions are expressed as virtual standing waves with $\pi/2$ phase. The frequency of the wave implies speed and time, corresponding to the propagation of the wave.

Propagation is interaction with itself (or with other?) and generates time according to special relativity :

$$\langle q|q \rangle = s^2 + x^2 + y^2 + z^2 = c^2 t^2 \quad \text{where } c \text{ is the speed of light}$$

Causality

The only thing that could change is the perspective of the observation, the projection on the observer's perspective. A projection is a removal of one or more dimensions. The change of perspective changes the direction of particles or creates particles or destroys particles in the observer's reference. A projection is an interaction. The non-associative bra-ket product introduces the causality by ordering operations, only if there is a scalar part in orthogonal quantions ($q = s + \vec{v}$, $s \neq 0$) because if $q = \vec{v}$, $q' = \vec{v}'$, $q'' = \vec{v}''$

$$\begin{aligned} \langle \langle \vec{v} | \vec{v}' \rangle | \vec{v}'' \rangle &= \langle (\vec{v} \cdot \vec{v}') - (\vec{v} \times \vec{v}') | \vec{v}'' \rangle = -(\vec{v} \cdot \vec{v}') \vec{v}'' + \vec{v} \times \vec{v}' \times \vec{v}'' \\ \langle \vec{v} | \langle \vec{v}' | \vec{v}'' \rangle \rangle &= \langle \vec{v} | (\vec{v}' \cdot \vec{v}'') - \vec{v}' \times \vec{v}'' \rangle = (\vec{v}' \cdot \vec{v}'') \vec{v} + \vec{v} \times \vec{v}' \times \vec{v}'' \end{aligned}$$

The scalar part introduces ability of mirroring without changing vectorial part, by changing sign of the scalar part.

Time

The scalar part generates time. The quantions as unitary quaternions are isomorph to SU(2) Lie group and a composition of SU(2) groups generates the standard model, including the Higgs boson.

- 1 $i\text{SU}(2) \times i\text{SU}(2) \times i\text{SU}(2) \subset U(3)$
- 2 $\dim(\text{SU}(2) \times \text{SU}(2) \times \text{SU}(2)) = 9 = \dim(U(3))$
- 3 from (1) and (2) : $U(3) = i\text{SU}(2) \times i\text{SU}(2) \times i\text{SU}(2)$
- 4 $U(3) = \text{SU}(3) \times U(1)$
- 5 from (3) and (4) : $\text{SU}(3) \times \text{SU}(2) \times U(1) \times i\text{SU}(2) = \text{SU}(2) \times \text{SU}(2) \times \text{SU}(2) \times \text{SU}(2) \times \text{SU}(2)$

The standard model $\text{SU}(3) \times \text{SU}(2) \times U(1)$ is based on a **broken** composition of $\text{SU}(2)$. The additionnal $i\text{SU}(2)$ in (5) can intuitively be seen as the 3 higgs bosons for the W and Z bosons, as well as the 3 higgs bosons for the 3 families of mass of fermions. The higgs bosons introduce a scalar (imaginary) part in the quantions.

Consequences

Field

The concept of field is immediate, it's the quantion's wave enveloping the whole universe.

Entanglement

The entanglement between two particles takes place after the interaction of the two particles, they share the same 3D space until the next interaction of one of the particles.

Antimatter

Matter and antimatter annihilation generates energy. Bra-ket multiplication of matter particle (q) with antimatter particle (q^*) has to generate vectorial particle ($\hat{v}v'$) in a collision ($v = v'$)

$$\begin{aligned} & \langle q|q^* \rangle = ss^* + v.v^* + \hat{i}(s^*v - sv^* - v \times v^*) = \hat{i}v' \neq 0 \\ \rightarrow & \quad ss^* + ||v||^2 = 0 \\ \rightarrow & \end{aligned}$$

Shape of the universe

Quantions are orthogonals in their interaction, with an euclidian norm, then the corresponding space-time structure is euclidian, so fundamentally flat.

Relativity

Theory agrees with the special relativity (Lorentz scalar) because of its fit with the Minkowski formula : $x^2+y^2+z^2+s^2 = c^2t^2$. Each elementary element, the quantion, has its own 3D reference space as required by the special relativity.

According to the above special relativity, gravity is generated by the 'mass' dimension (s) that decreases space (x,y,z) for same time (t), which agrees with the slowing down because of the mass in the general relativity. The general relativity is statistically generated from quantions and is not a fundamental structure of space-time. The number of quantions influences the behaviour of gravitation.

Unitary electric charge

If everything looks like a rotation of $\pi/2$ phase, the negative electric charge can be defined by a phase shift of $-\pi/2$ (charge -1). **Unit electric charges** could be used for up quark with shift of $2\pi/2$ (charge +2) and down quark with shift of $\pi/2$ (charge +1) because $4\pi/2 \equiv 0$.

$$\begin{aligned} \bullet \quad \text{up+up+down} &= (2+2+1) \pi/2 \equiv 1 \pi/2 & \text{(proton)} \\ \bullet \quad \text{up+down+down} &= (2+1+1) \pi/2 \equiv 0 \pi/2 & \text{(neutron)} \end{aligned}$$

By this way, a symmetry could be established between the charge of leptons (electron -1, neutrino 0) and the one of quarks (down +1, up +2).

Conclusion

Based on the hope that Nature is simple, this article introduces a new representation of space-time structure of the universe : an hypersphere structure on a multi-dimensional space, each dimension is an energy quantum with its opposite forming a quaternion covering the whole universe. The

implications of this hypothesis are vast and go far beyond this short article.

There is still a long way to envolve the whole physic in one theory but this bottom-up approach, from simple principles to more complex structures, in adequation with the observed reality, is probably a good way to elaborate a simple and comprehensive theory. This intuitive approach tries to answer to a fundamental question : why has the universe an apparent 3 dimensional structure in addition of time, which is far from an evidence ?

Whether the theory is correct or not, it seems increasingly clear that the space-time is not a fundamental structure, it's the consequence of the interaction between particles. That's why calculations based on space-time can become unstable. To explain the universe, the ether is not necessary, perhaps neither is space-time.

Puzzle :

- Clé :
 - multiplication : prq ?
 - conservation du moment : même référentiel 3D,
 - interaction : propagation ok mais pas distances, impossible si ponctuel (\rightarrow aléatoire),
 - fermion : pas d'inverse, masse imaginaire,
- Un scalaire imaginaire est un bi-spineur de Dirac :
 - vérifier que le scalaire complexe est suffisant (coordonnées pas imaginaires)
 - lien avec anti-particule (négatif)
 - indispensable à l'interaction (principe exclusion) et donc à l'écoulement du temps.
- Interaction
 - lié à un opérateur hermitien, c'est une projection (= rotation) qui conserve la quantité de mouvement (direction conservée, dérivée par rapport à direction, énergie/c)
 - Ponctuelle \rightarrow crée l'espace
 - La probabilité serait-elle le résultat de l'interaction et non l'inverse ?
 - Un spin « orienté » garde son orientation, c'est une contrainte dans l'orientation générale (génération 3D)
 - probabilité nécessaire sinon relativité restreinte violée (effondrement)
 - dépend de la longueur d'onde
 - dépend du principe d'exclusion
 - rotation des probabilités sans forcément se réaliser \rightarrow probabilité devient 100 %
 - projection \rightarrow produit
 - division (braket)
 - entropie,
 - plus possible (entropie maximale) si info aléatoire
 - impossible si division par 0
 - espace 3D global à cause de la brisure de symétrie (référence globale)
 - notion de distance et d'ordre (pas hypersphère indépendante)
- propagation
 - à partir de l'onde stationnaire ?
 - = Module
 - auto-interaction
 - loi des grands nombres
 - phase sans importance, seule compte les probabilités
 - unitaire (temps unitaire car autre côté équation)
 - imprécision de la vitesse (c pourtant) et de la position à cause de l'imprécision de l'espace/temps
 - propagation depuis position « précise »
- Axe réel = énergie cinétique, axe imaginaire = énergie potentielle
- utiliser sa racine carrée si une particule ne semble pas respecter les règles
- Particule droite/gauche selon isospin nul ou non
- neutrino : pseudovecteur (produit vectoriel), pourquoi pas de charge, =2 ?
- $E=mc^2$, pourquoi ici ?
- La vitesse augmente la masse
- Brisure de symétrie : $SU2 \times U1 \rightarrow SU2 + U1$ (? bizarre)
- orthogonalité = indiscernabilité, décohérence,
- contradiction entre aléatoire et loi des grands nombres